The uniform-matroid finite multiplier sequence: source-v1

Myeongjin Kang

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Research implementation manuscript, 2026-11-15 UTC. MathDB339675.

Source and statement

Gao, Lu, Xie, Yang and Zhang, The Kazhdan-Lusztig polynomials of uniform matroids,
owned journal-associated 2021 body, Section 3.2, Eq. (3.3), Conjecture 3.3
(p.20). The source family is indexed by positive integers m,d. The normalized
MathDB line is target-only evidence, not a fresh canonical observation. Source references:
https://arxiv.org/abs/1806.10852 and
https://mathdb.com/p/339675/the-d-sequence-conjecture-for-coefficients-of-uniform-matroi . Only owned supplied source scope is claimed, not final-journal identity beyond it.

For real x and integer h>=0 let C(x,h)=product_{k=0}^{h-1}(x-k)/h!. Define
b_m(d,x)=sum_{h=0}^{m-1} [m+x(h-m+1)]/[m(h+1)]
C(x-1+h,h) C(d-x+h,h). All subtraction is in R. In particular C(-1,0)=1; no truncated natural subtraction
is used. Let R(t)=sum_{i=0}^d choose(d,i)b_m(d,i)t^i. Theorem: for every positive integer m,d and every real polynomial
F(t)=sum c_i t^i of degree at most d with only real zeros,
sum_{i=0}^d b_m(d,i)c_i t^i is zero or has only real zeros. The zero polynomial is explicitly included. Mixed signs, zero roots, repeated roots
and lower degrees impose no extra hypotheses. This is the full source d-sequence
preservation assertion. No necessity direction of a finite criterion is needed.

Ordinary proof: the exterior-root polynomial

Fix positive m,d; a_j=j(d+j), K_j=(-1)^j choose(d+j-1,j). The key identity, including j=m, is
p(-j)=p(d+j)=K_j(1-a_j/(md)), 0<=j<=m. (E)
Here is a full symbolic derivation, not a computational oracle. For positive A>=q+1, finite differences of Pascal give
sum(h=0..q)(-1)^h choose(q,h)choose(A+h,h)=(-1)^q choose(A,q);
sum(h=0..q)(-1)^h choose(q,h)choose(A+h,h)/(h+1)
=(-1)^q choose(A-1,q)/(q+1). For the second, replace the last quotient by choose(A+h,A-1)/A and
take q differences; choose(A-1,q)/ (q+1) results by factorial cancellation. All applications below have positive integral A and no factorial poles.

p(0)=1. At d its summand is
[d/m-(d-1)/(h+1)] choose(d+h-1,h). For d>1 the subtracted product is choose(d+h-1,h+1);
hockey-stick summation leaves 1 since
d/m choose(d+m-1,m-1)=choose(d+m-1,m). For d=1 each of m summands is1/m. Thus p(d)=1.

At -j, 1<=j<=m, first binomial is (-1)^h choose(j,h), vanishing for h>j. At j=m the extra h=m term can be inserted because its linear numerator is0. The prefactor becomes (j+1)/(h+1)-j/m. The two difference identities with q=j,A=d+j give (-1)^j[choose(d+j-1,j)-(j/m)choose(d+j,j)], exactly(E). At d+j, second binomial is (-1)^h choose(j-1,h), vanishing for h>=j. The prefactor is (1-d-j)/(h+1)+(d+j)/m. Use q=j-1,A=d+j-1. The result is
(-1)^(j-1)[(1-d-j)/j choose(d+j-2,j-1)
+(d+j)/m choose(d+j-1,j-1)]. The first binomial multiple is -choose(d+j-1,j);
the second equals a_j/(md)choose(d+j-1,j). This proves(E).

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Degree: h<m-1 terms have degree<=2m-3; last prefactor is1/m. Thus p has exact degree2m-2 and leading coefficient (-1)^(m-1)/[(m-1)!m!]. For m=1 it is1. Interpolate B(s), degree<=m-1, at s=-a_j for j=0..m-1 using(E). The polynomials p(u), B(u(d-u)) agree at 2m distinct points
-j,d+j (0<=j<=m-1), and both have degree<=2m-2. They coincide identically. Therefore B has exact degree m-1, leading coefficient1/[(m-1)!m!], B(0)=1,
and B(-a_j)=K_j(1-a_j/(md)) ALSO at the extra j=m.

Root count: 0=a_0<...<a_m and 0<md<a_m. If a_l<md<a_(l+1),
the signs at nodes are (-1)^j for j<=l, and (-1)^(j+1) for j>=l+1. Among m adjacent node pairs exactly one fails to alternate. IVT gives m-1 distinct negative roots in disjoint open intervals. If md=a_l, necessarily1<=l<=m-1. There are l-1 roots on the left,
m-l-1 on the right, and the exact node root -a_l: again m-1 distinct
negative roots. Degree excludes every additional real/complex root and
all multiplicities. For m=1 the statement is vacuous. Hence B(s)=product_r(1+s/beta_r), beta_r>0.

Each factor lifts to roots
(d-sqrt(d^2+4 beta_r))/2<0 and (d+sqrt(d^2+4 beta_r))/2>d
of p(u). So p has only exterior roots and p(i)>0 on0<=i<=d.

Ordinary proof: from exterior roots to the literal symbol
Write theta=tD. For a degree-d real polynomial F with strictly negative real roots
rho_j (listed with multiplicity), at Im z>0,
theta F(z)/F(z)=sum_j z/(z-rho_j). The imaginary part equals sum_j (-rho_j) Im(z)/|z-rho_j|^2, hence is positive when d>0. Thus (theta-r)F has no upper half-plane zero for any real r;
conjugation excludes the lower half-plane too. For t>0 its logarithmic quotient
is a sum of d numbers strictly between 0 and 1. If r<0 or r>d there are no
positive zeros. The constant coefficient is multiplied by -r and the leading
coefficient by d-r, both nonzero. Therefore (theta-r) preserves exact degree d
and strictly negative real zeros, without a simplicity assumption. Factor p over its exterior real roots, apply these operators successively, and
include the nonzero leading scalar. Starting with (1+t)^d gives p(theta)(1+t)^d. On monomials theta acts by i, so its ith coefficient is choose(d,i)p(i). Consequently it equals the actual R, not an alias or an ordinary Z-polynomial. This proves R splits over R and all its real zeros are negative. The endpoint evaluations p(0)=p(d)=1 give constant coefficient 1 and exact degree d.

Ordinary proof: finite Schur composition for all real inputs
Here is the sufficient Schur argument, whose steps are also in the Lean closure. Call a complex polynomial stable if it has no zeros in the open upper half-plane. For a stable nonconstant P with degree k<=n and Im z=y>0 put S=P'(z)/P(z)=sum_j 1/(z-r_j), where Im r_j<=0. Then -Im S>=y sum_j |1/(z-r_j)|^2, and Cauchy-Schwarz yields y|S|^2<=n(-Im S). In particular S is nonzero. For Im w>0 the polynomial P+(w-t)P'/n is stable: a zero at z
would give w=z-n/S, whose imaginary part is
y+n Im S/|S|^2<=0, a contradiction. Constants are immediate. Its degree is at most n-1. Iterate this operation with n=d,d-1,...,1. The resulting symmetric multiaffine polarization is nonzero when all its d
arguments lie in the upper half-plane. Expanding on monomials identifies it as
Pol_d(F)(z_1,...,z_d)=sum_i c_i e_i(z_1,...,z_d)/choose(d,i). This follows either by induction on the steps or by symmetry and its diagonal
value F(t): each symmetric multiaffine homogeneous term is a scalar multiple
of e_i, and e_i(t,...,t)=choose(d,i)t^i.

Since R has exact degree d, constant 1, and strictly negative roots, write
R(t)=R(0) product_j(1+t/s_j), s_j>0. For every upper half-plane z, the numbers z/s_j are in that half-plane. The weighted composition

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W_d(F,R)(t)=sum_i c_i R.coeff(i)t^i/choose(d,i)
equals R(0) Pol_d(F)(t/s_1,...,t/s_d), so is stable for nonzero real-rooted F. It has real coefficients; conjugation implies all its complex roots are real,
hence it splits over R. For F=0 the output is zero directly. Finally choose(d,i)>0 for 0<=i<=d, so cancellation gives exactly
W_d(F,R)=sum_i c_i b_m(d,i)t^i. This proves the theorem.

Formal realization and fidelity
Namespace MathDB339675. The root is UMWFinalSourceAssemblyV1. SourceTarget is forall m d:Nat, 0<m -> 0<d -> IsDSequence m d. The theorem source_target has exactly this type.
source_target_literal expands IsDSequence, diagonal, b and fallingChoose into
finite sums/products. It is proved by the same source_target term through
definitional equality, rather than imported theorem text or a hypothesized bridge.
source_theta_seed_eq_symbol is a coefficient equality using eval_sourceP;
composition_symbol_eq_diagonal is another actual polynomial extensionality proof. Generic finite_schur_splits accepts arbitrary zero-or-split real inputs.

JOINT's common source definitions and weighted composition remain the sole owners. The nine affected SYMBOL modules have AssemblyV1 names and fresh compiled objects. No alternate Slim owners were imported. All changed files were individually
compiled and independently kernel checked. Only propext, Classical.choice and
Quot.sound occur in the transitive endpoint audit. In particular Schur and the
theta preservation argument are proved in the closure, not added axioms. The appendix is the complete actual coherent production closure, in dependency
order. It contains no conflicting obsolete providers or failed proof drafts.

Bounded contribution assessment (inherited scoped evidence)
2021 author-posted journal-associated body: Section3.2 Eq3.3 is exactly p. Theorem3.4 treats2<=m<=15, all d. Its proof uses falling factorial expansion
and Hurwitz determinants, displays m=2 and refers m=3..15 computations to an
author repository. That code is neither acquired nor executed. Printed Eq3.6
says “nonnegative” zeros; the sign typo is not a contribution.
2021 Eq3.2 sends R to Z by the extra diagonal
a_i=choose(d+2m,i+m)/choose(d+2m,m). This forward preserver has no supplied
inverse-preservation theorem. General real-rootedness of ordinary Z cannot
silently establish the target. Source theorem numbers are version-specific.

2024 Wu–Xie–Zhang Theorem3 Eq2 gives GAMMA coefficients for1<=i<=floor(d/2):
r_i=d!/[i!(d-2i)!(i-1)!] sum(h=0..m-1)choose(d+h,h)/[(h+i)(h+i+1)]. Personally read its printed pp13–14 proof: Eq21, m=1 initialization and
forward difference in m, with printed Gosper/Mathematica/Annihilator steps. No CAS replay is claimed. Its ultra log-concavity does not imply real-rootedness,
much less the inverse-diagonal d-sequence claim.

Owned2609.07636v1: personally read Proposition2.1 and proof, Proposition3.2
through its interpolation conclusion, and the complete Lemma4.1 proof. Its
N(y)=sum(j=0..m-1)(m-j)!choose(m+d,j)(y)_{j,rising} acts on gamma grid
0..floor(d/2), with coefficient d!N(i)/[(d-2i)!i!(i+m)!]. Proposition3.2 uses Chu–Vandermonde for
N(-j)=(-1)^j(m-j)!(d)_{j,rising}. Lemma4.1 interpolates at equally spaced integer nodes; its rational
representation has strictly positive residues, negative derivative and one
root in every gap. Its shifted-d comparison also uses residue positivity. This is the strongest contrary case: the

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candidate uses the SAME classical
finite-sum evaluation, interpolation and sign-count strategy. Novelty of these
tools, of Schur, or of the generic IVT step is not claimed.

A complete technical specialization of Lemma4.1 to B has NOT been established:
substituting y or simply changing node names does not change N into p or B. B has quadratic nodes, an extra endpoint, residues proportional to
1-a_j/(md), of both signs, sometimes exactly zero; the all-positive-residue
hypothesis and global monotonicity in Lemma4.1 fail. The specific additional argument is(E), paired interpolation identifying the
quadratic quotient, and the omitted-gap/equality root count above. For the strongest direct method adaptation, set P(s)=product(j=0..m)(s+a_j). P'(-a_j)=(-1)^j D_j, D_j=j!(m-j)! product(l!=j)(d+j+l)>0. Then B/P=sum_j c_j(1-a_j/(md))/(s+a_j), c_j=choose(d+j-1,j)/D_j>0,
by interpolation. There is no polynomial part. A zero residue is a canceled
pole, not a reason to delete a root count. Since degB=m-1 and degP=m+1,
the residue sum is0, making an “all residues positive” transplant impossible. The preceding IVT count is the full repair needed for this family, not a
consequence of the source monotonicity claim.

Bounded scientific judgment: SUBSTANTIAL candidate, because the explicit
uniform evaluation/cancellation mechanism closes the full source conjecture
beyond the published fixed-m determinant method and is not supplied by the
read source theorem/proof passages. This judgment is based on the actual
additional argument, not on proof length or failure to find a published exact
theorem. A later adjudicator could deem this a technical method adaptation;
the strongest case for that is precisely the common Chu–Vandermonde/interpolation
architecture. No worldwide absence or originality certification follows. No inaccessible protected scientific body was reconstructed.

Validation and limits
Twenty formal calls (10 compiles and 10 kernel checks), no formal failures. No network requests, searches, scientific prototypes, children or shared count edits. Unchanged JOINT source/olean pairs were reused only under the explicit owned
artifact grant, with input hashes verified and copied output bytes checked. This is not an independent fresh whole-closure replay; that remains for root
acceptance. Formal receipts and pins are in FINAL-BUILD-GRAPH-v1.json. No statement about worldwide absence or journal-final fidelity follows. All historical process defects remain unwaived; see PROCESS-INVENTORY-v1.md.

Literal source appendix

The complete literal source appendix is supplied byte-exact in publication/complete-literal-source-appendix.md in supporting.zip and proof-sources.json. Extract using extract_sources.py. No ordinary proof above this appendix has been removed.

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