A Rank-Four Binary Counterexample to the Lasoń Common-Basis Conjecture (MathDB 332371)

Myeongjin Kang

Version 2, November 17, 2026 (version 1: November 7, 2026)

Version 2. This version corrects the explanation in one worked membership instance in Section 4: version 1 justified $\{0,2,3,5\} \notin D_6$ by the false assertion $6 \in \{0,2,3,5\}$. The conclusion is correct for a different reason, now given: the complement $\{1,4,7,8\}$ is dependent because elements 1 and 4 have equal columns. The counterexample, all tables, the theorem, its proof and the exhaustive obstruction are unchanged. Internal review-status annotations of version 1 are replaced by the accurate statements of Sections 6 and 8 and of the note below; the full list of changes is in the supplementary CHANGELOG.md.

AI assistance and review. This is an AI-assisted research preprint. The witness search, verification scripts, Lean development, literature comparison and manuscript drafting were carried out with AI assistance under the author's direction. The reviews cited below are internal AI-conducted reviews; they are not external journal peer review, and bounded prior-art searches do not certify worldwide priority.

mathdb:2611.00061v2 17 Nov 2026

Contribution. An explicit rank-4 matroid on the nine-element ground set $E = \{0,1,...,8\}$ such that, with $x = 1$ and $y = 6$, both $E\backslash x$ and $E\backslash y$ partition into two bases, but no two such partitions share a base. This refutes, in the exact printed scope, Conjecture 2.5 of Lasoń (Selecta Mathematica 2021) = Conjecture 13 (arXiv v2/v3), the common-basis conjecture for partitions of a matroid ground set, for $k = 2$, $r = 4$.

1. The conjecture

Throughout, matroids are finite matroids in full generality; no simplicity, representability, or connectedness is assumed. The following is Conjecture 2.5 in the journal version and arXiv v4/v5 of [1]; it appears as Conjecture 13 with the same quantifiers in arXiv v2/v3, and v1 predates the formulation.

> **Conjecture 2.5 ([1], journal form).** Let $k \geq 2$, and let M be a matroid of rank r on a ground set E with $|E| = kr+1$. Suppose $x, y \in E$ are such that both $E\backslash\{x\}$ and $E\backslash\{y\}$ can be partitioned into k pairwise disjoint bases. Then there exist partitions of $E\backslash\{x\}$ and $E\backslash\{y\}$ into k pairwise disjoint bases which share a common basis.

The source derives the statement for classes of matroids closed under rank-preserving deletions and parallel extensions, and records (attributing the history to Joseph Bonin) that the case $k = 2$ was studied by Paul Seymour and Neil White in the 1980s and was not resolved. Proposition 2.6 of [1] proves one-way implications between this conjecture, the complementary-basis-graph conjecture, and White's conjecture; Proposition 2.7 proves the conjecture for $k \geq 2^{r-1}+1$, which is $k \geq 9$ at $r = 4$ and so does not cover $k = 2$. The implications in Proposition 2.6 are one-way: a counterexample to Conjecture 2.5 is not a counterexample to any White conjecture, and we claim none.

2. The counterexample

Let $E = \{0, 1, ..., 8\}$, $x = 1$, $y = 6$, $r = 4$, $k = 2$, so $|E| = 2\cdot4+1$ as required. Consider the binary representation whose columns are the integers below, read as four binary coordinates (b3 b2 b1 b0, i.e. coordinate i is the coefficient of 2^i):

element	0	1	2	3	4	5	6	7	8
column integer	9	6	4	11	6	14	5	1	12
b3 (coefficient of 8)	1	0	0	1	0	1	0	0	1
b2 (coefficient of 4)	0	1	1	0	1	1	1	0	1
b1 (coefficient of 2)	0	1	0	1	1	1	0	0	0
b0 (coefficient of 1)	1	0	0	1	0	0	1	1	0

Table 1. The rank-4 binary representation; column j is the integer in the second row, in binary.

A four-element subset of E is a base exactly when the corresponding 4×4 determinant is odd. Of the C(9,4) = 126 four-subsets, exactly **76** are bases; the complete list follows.

{0, 1, 2, 5}	{0, 1, 2, 6}	{0, 1, 2, 7}	{0, 1, 2, 8}
{0, 1, 3, 5}	{0, 1, 3, 6}	{0, 1, 3, 7}	{0, 1, 3, 8}
{0, 1, 5, 6}	{0, 1, 5, 8}	{0, 1, 6, 7}	{0, 1, 7, 8}
{0, 2, 3, 5}	{0, 2, 3, 6}	{0, 2, 3, 7}	{0, 2, 3, 8}
{0, 2, 4, 5}	{0, 2, 4, 6}	{0, 2, 4, 7}	{0, 2, 4, 8}
{0, 2, 5, 6}	{0, 2, 5, 7}	{0, 2, 5, 8}	{0, 3, 4, 5}
{0, 3, 4, 6}	{0, 3, 4, 7}	{0, 3, 4, 8}	{0, 3, 5, 7}
{0, 3, 6, 7}	{0, 3, 7, 8}	{0, 4, 5, 6}	{0, 4, 5, 8}
{0, 4, 6, 7}	{0, 4, 7, 8}	{0, 5, 6, 7}	{0, 5, 7, 8}
{1, 2, 3, 5}	{1, 2, 3, 6}	{1, 2, 3, 7}	{1, 2, 3, 8}
{1, 2, 5, 6}	{1, 2, 5, 7}	{1, 2, 6, 8}	{1, 2, 7, 8}
{1, 3, 5, 7}	{1, 3, 5, 8}	{1, 3, 6, 7}	{1, 3, 6, 8}
{1, 5, 6, 7}	{1, 5, 6, 8}	{1, 5, 7, 8}	{1, 6, 7, 8}
{2, 3, 4, 5}	{2, 3, 4, 6}	{2, 3, 4, 7}	{2, 3, 4, 8}
{2, 3, 5, 8}	{2, 3, 6, 8}	{2, 3, 7, 8}	{2, 4, 5, 6}
{2, 4, 5, 7}	{2, 4, 6, 8}	{2, 4, 7, 8}	{2, 5, 6, 8}
{2, 5, 7, 8}	{3, 4, 5, 7}	{3, 4, 5, 8}	{3, 4, 6, 7}
{3, 4, 6, 8}	{3, 5, 7, 8}	{3, 6, 7, 8}	{4, 5, 6, 7}
{4, 5, 6, 8}	{4, 5, 7, 8}	{4, 6, 7, 8}	{5, 6, 7, 8}

Table 2. The 76 bases of M, the complete base family used throughout; it is also the literal Lean certificate.

3. Verification of the axioms, without the representation

four-element subsets of E	C(9,4) = 126
bases (Table 2)	76
ordered exchange obligations (A, B, e)	12,736
exchange failures	0
circuits	23: one parallel pair, three triangles, eleven 4-circuits, eight 5-circuits
bases extendible for z = 1 / z = 6	|D1| = 32 / |D6| = 24
unordered partitions of E\{1} / E\{6}	16 / 12
bases shared by D1 and D6	0

Table 3. Complete counts for the witness; every entry is rederived from the certificate in the supplementary archive.

The base family of Table 2 satisfies the basis-exchange axiom directly, so M is a matroid independently of the binary model; the Lean endpoint indeed uses only the list. Membership was decided by parity of the 4×4 determinant (equivalently, Gaussian elimination over F2 on the four coordinates of Table 1; the two procedures agree on all 126 subsets). For every ordered pair of bases A, B and every e ∈ A\B - 12,736 triples in all - a witness f ∈ B\A with (A\{e})∪{f} a base exists; the independent recheck performed for this manuscript found no failure. Nonempty proper bases of equal size give a rank-4 matroid on E; the rank is witnessed concretely, e.g. {0, 2, 3, 5} is a base, so r = 4.

Worked exchange instance. A = {0,2,3,5} and B = {0,1,2,6} are bases and e = 3 lies in A\B; the element f = 1 of B\A gives (A\{3})∪{1} = {0,1,2,5}, a base of Table 2 (f = 6 works too). Every one of the 12,736 triples is disposed of in this way by the certificate in the supplementary archive and by the kernel-checked Lean obligations of Section 6.

Ordinary structural facts (supplementary; computed but not part of the formal endpoint): the 23 circuits are one parallel pair {1, 4}, three triangles {0,6,8}, {2,6,7}, {3,5,6}, eleven 4-circuits and eight 5-circuits. Every element lies in a base (no loops), and every two elements lie in a common circuit, so M is connected. Elements 1 and 4 are

mathdb:2611.00061v2 17 Nov 2026

parallel; the source setting explicitly permits parallel elements (its closure hypotheses require parallel extensions), and no printed simplicity restriction exists. The distinguished elements x = 1 and y = 6 are distinct labels; deletion partitions in any case force x and y to be nonloops.

4. Two partitions and the complete common-base obstruction

Both required partitions exist and are displayed by the theory:

Partition of E\{1}. E\{1} = {0,2,3,5} disjoint-union {4,6,7,8}.

Partition of E\{6}. E\{6} = {0,1,2,7} disjoint-union {3,4,5,8}.

For $z \in \{1, 6\}$ let $D_z = \{ B \text{ base} : z \notin B \text{ and } (E\backslash\{z\})\backslash B \text{ is a base} \}$, the bases extendible to a two-base partition of E\{z}. A common base of two partitions of E\{1} and E\{6} must avoid 1 and 6 and hence lie in $D_1 \cap D_6$ with both complements bases; conversely any $B \in D_1 \cap D_6$ yields shared-base partitions. So the obstruction is exactly $D_1 \cap D_6 = \emptyset$, checked on the complete lists below - not merely on the two displayed partitions.

base B	complement base	base B	complement base
{0, 2, 3, 5}	{4, 6, 7, 8}	{2, 3, 4, 6}	{0, 5, 7, 8}
{0, 2, 3, 6}	{4, 5, 7, 8}	{2, 3, 4, 8}	{0, 5, 6, 7}
{0, 2, 3, 7}	{4, 5, 6, 8}	{2, 3, 5, 8}	{0, 4, 6, 7}
{0, 2, 3, 8}	{4, 5, 6, 7}	{2, 3, 7, 8}	{0, 4, 5, 6}
{0, 2, 4, 5}	{3, 6, 7, 8}	{2, 4, 5, 6}	{0, 3, 7, 8}
{0, 2, 4, 6}	{3, 5, 7, 8}	{2, 4, 6, 8}	{0, 3, 5, 7}
{0, 2, 5, 7}	{3, 4, 6, 8}	{2, 5, 6, 8}	{0, 3, 4, 7}
{0, 2, 5, 8}	{3, 4, 6, 7}	{2, 5, 7, 8}	{0, 3, 4, 6}
{0, 3, 4, 6}	{2, 5, 7, 8}	{3, 4, 6, 7}	{0, 2, 5, 8}
{0, 3, 4, 7}	{2, 5, 6, 8}	{3, 4, 6, 8}	{0, 2, 5, 7}
{0, 3, 5, 7}	{2, 4, 6, 8}	{3, 5, 7, 8}	{0, 2, 4, 6}
{0, 3, 7, 8}	{2, 4, 5, 6}	{3, 6, 7, 8}	{0, 2, 4, 5}
{0, 4, 5, 6}	{2, 3, 7, 8}	{4, 5, 6, 7}	{0, 2, 3, 8}
{0, 4, 6, 7}	{2, 3, 5, 8}	{4, 5, 6, 8}	{0, 2, 3, 7}
{0, 5, 6, 7}	{2, 3, 4, 8}	{4, 5, 7, 8}	{0, 2, 3, 6}
{0, 5, 7, 8}	{2, 3, 4, 6}	{4, 6, 7, 8}	{0, 2, 3, 5}

Table 4. D1: the 32 bases extendible to a partition of E\{1} (32 give 16 unordered partitions).

base B	complement base	base B	complement base
{0, 1, 2, 7}	{3, 4, 5, 8}	{1, 2, 3, 5}	{0, 4, 7, 8}
{0, 1, 2, 8}	{3, 4, 5, 7}	{1, 2, 3, 7}	{0, 4, 5, 8}
{0, 1, 3, 5}	{2, 4, 7, 8}	{1, 2, 5, 7}	{0, 3, 4, 8}
{0, 1, 3, 8}	{2, 4, 5, 7}	{1, 2, 7, 8}	{0, 3, 4, 5}
{0, 1, 5, 8}	{2, 3, 4, 7}	{1, 3, 5, 7}	{0, 2, 4, 8}
{0, 1, 7, 8}	{2, 3, 4, 5}	{1, 3, 5, 8}	{0, 2, 4, 7}
{0, 2, 4, 7}	{1, 3, 5, 8}	{2, 3, 4, 5}	{0, 1, 7, 8}
{0, 2, 4, 8}	{1, 3, 5, 7}	{2, 3, 4, 7}	{0, 1, 5, 8}
{0, 3, 4, 5}	{1, 2, 7, 8}	{2, 4, 5, 7}	{0, 1, 3, 8}
{0, 3, 4, 8}	{1, 2, 5, 7}	{2, 4, 7, 8}	{0, 1, 3, 5}
{0, 4, 5, 8}	{1, 2, 3, 7}	{3, 4, 5, 7}	{0, 1, 2, 8}
{0, 4, 7, 8}	{1, 2, 3, 5}	{3, 4, 5, 8}	{0, 1, 2, 7}

Table 5. D6: the 24 bases extendible to a partition of E\{6} (24 give 12 unordered partitions).

Worked membership instances. The base B = {0,2,3,5} lies in D1: it avoids 1, and the complement (E\{1})\B = {4,6,7,8} is a base (row 1 of the partition above). The base {0,2,3,6} also lies in D1, its complement {4,5,7,8} being a base; but {0,2,3,5} does not lie in D6. Although it avoids 6, its complement in E\{6} is {1,4,7,8}, and elements 1 and 4 have the identical nonzero column 6 in Table 1; equal columns give a nonzero dependence over F_2 among these four columns, so this complement is not a base. Checking all 76 bases in this way produces Tables 4 and 5 exactly.

mathdb:2611.00061v2 17 Nov 2026

Proposition (complete finite obstruction). $D_1 \cap D_6 = \emptyset$. Equivalently, there do not exist partitions of E\{1} and E\{6} into two bases each sharing a common base.

Proof. Every member of D_1 avoids 1 and every member of D_6 avoids 6; a shared base B would lie in both lists. The lists of Tables 4 and 5, computed by enumerating all 76 bases and testing the complement, share no member (the intersection of the two displayed lists is empty), and each list is exhaustive by construction. Applied to the concrete complements: for B in D1 the element set (E\{1})\B must be one of the 16 complementary bases, and simultaneously (E\{6})\B one of 12; the joint condition fails for all 32 candidates. □

Each D-list is closed under complement-pairing, so $|D_1| = 32$ gives 16 unordered partitions of E\{1} and $|D_6| = 24$ gives 12 of E\{6}; Tables 4 and 5 enumerate all of them through their first halves, the second half being the swapped pairs.

Negation of the named instance. In Conjecture 2.5 of [1] (Conjecture 13 of arXiv v2/v3), take k = 2, r = 4, E = {0,...,8}, x = 1, y = 6. The hypothesis holds: $|E| = 2 \cdot 4+1$ and both E\{1} and E\{6} partition into two disjoint bases. The conclusion fails: there do not exist partitions of E\{1} and E\{6} into two pairwise disjoint bases sharing a common base. This instance-level failure is exactly the negation of the printed conjecture for these parameters; no broader claim is made.

5. Why naive shortcuts fail (remarks)

The families D_1 and D_6 are not themselves matroid base families: in D_1 the bases A = {2,5,6,8}, B = {3,5,7,8} with e = 2 have no eligible exchange inside D_1, and in D_6 the same failure hits A = {0,4,7,8}, B = {0,3,4,5} with e = 8. The two-matroid intersection theorem therefore does not apply to D_1, D_6 directly. On F = E\{1,6} a desired B would be a common base of three structures at once: the restriction M|F and the two single-element deletions of duals, ((M\1)*)\6 and ((M\6)*)\1, each of rank 4 on F; the inspected common-base results for two matroids do not settle this triple condition. These are explanations of failed shortcuts, not a general non-reducibility claim.

The source's Proposition 2.7 gives the property for $k \geq 2^{r-1}+1$; at r = 4 the threshold is 9, and our k = 2 lies outside every case it covers. No printed construction, figure, or formula in any version of [1] produces this witness; it was found by a bounded random search over rank-4 binary representations on nine points and then verified exactly.

6. Formal verification in Lean

The literal Lean endpoint is the theorem `CommonBasis332371.counterexample` in a twelve-module development over Lean 4.34.1 and mathlib `d13f23b723b8a846827a245b89c10fc7d3f11612` (pinned; manifest in the supplementary archive). Its statement is:

```
∃ N : Matroid (Fin 9), N.E = univ ∧ N.E.ncard = 2*4+1 ∧ N.eRank = 4 ∧ (∃ A B, TwoBasePartition N (N.E \ {1}) A B)
∧ (∃ A B, TwoBasePartition N (N.E \ {6}) A B) ∧ ¬(∃ B C D, TwoBasePartition N (N.E \ {1}) B C ∧
TwoBasePartition N (N.E \ {6}) B D)
```

Here TwoBasePartition N S A B asserts that A and B are *actual* N-bases (Mathlib IsBase), disjoint, with union exactly S. The development constructs N by `Matroid.ofIsBaseOfFinite` from the explicit list of Table 2 plus kernel-checked exchange obligations: modules `Bases` (the 76-entry list and the base predicate), `Cases0`-`Cases7` (all finite checks by decide), `Endpoint` (the exchange property and the matroid M), `Final` (both partitions, the no-shared-base direction via complement characterization, ground cardinality and rank), and `Audit`. Because the base family and its exchange verification construct a genuine Mathlib Matroid, no binary-representation bridge is formalized or needed. The recorded transitive axiom footprint of the endpoint is exactly propext, Classical.choice, Quot.sound; the in-development metadata audit covers the 106 declarations of the CommonBasis332371 namespace. No sorry, admit, added axioms, unsafe code, or native_decide is used. The certificate is the full literal statement of the counterexample: it formalizes the matroid, both partitions and the complete no-shared-base conclusion, not a proxy.

Evidence status. The following checks were recorded on November 5, 2026, before version 1 was released; no Lean replay was rerun for version 2. An author-side strict replay compiled all 12 modules and checked them with

mathdb:2611.00061v2 17 Nov 2026

leanchecker. An independent AI-conducted Lean-fidelity review passed its gates for literal source fidelity, mathematical semantics, strict replay (24/24 stages: lean and leanchecker for each module), package scope and documented reproducibility limits, including a body-presence audit of the 106 namespace declarations, the exact propext/Classical.choice/Quot.sound axiom closure, and negative controls. A supplementary audit selected declarations by originating module and found 107: the 106 namespace declarations plus the Lean-generated congruence lemma Matroid.ofIsBaseOfFinite.congr_simp. It printed all 107 types in full and confirmed bodies and the same axiom closure for each. A final replay under the enforced wrapper of Section 8 again passed all 24 strict stages. Trust limits: the prebuilt upstream mathlib packages are trusted without a source-to-olean attestation, and leanchecker shares the Lean installation. The receipts are in the supplementary archive. Sections 2-5 are ordinary finite mathematics; this section is formal bookkeeping.

`Bases.lean`	the 76-entry base list, the base predicate BP, the Finset form
`Cases0.lean - Cases7.lean`	the 76 kernel decide-checks, one per base, of all exchange and complement conditions
`Endpoint.lean`	transfer to Matroid.ExchangeProperty BP; the matroid M by Matroid.ofIsBaseOfFinite
`Final.lean`	TwoBasePartition, both partitions, no-shared-base, the literal counterexample
`Audit.lean`	per-declaration axiom and metadata audit over the 106 CommonBasis332371 namespace declarations

Table 6. The twelve Lean modules (Cases0-Cases7 are eight files) and their roles; SHA-256 manifest in the supplementary archive.

7. Novelty, significance, and limits

Significance: the conjecture is explicitly printed, with identical quantifiers, in arXiv versions v2-v5, the journal, and an independent 2016 workshop problem list; the case k = 2 was posed in the 1980s (Seymour-White, as reported in [1]) and left open; and the refutation is a complete finite obstruction requiring no representability. Novelty limits, per the independent AI-conducted literature review: the underlying rank-4 matroid on nine points is not claimed new; no minimality of nine elements or rank four is claimed; no simple-matroid or infinite-family strengthening is claimed; and no worldwide priority is asserted.

Relation to prior negative results. Larson [2] (July 2026) gives, in Example 1.4 and Theorem 1.5, a rank-9 binary matroid on 18 elements whose complementary-basis reconfiguration fails, via a split-pair invariant taking values 3, 7, 9 and changing by at most 2; this refutes Conjectures 1.1(2) and 1.2 there, a White-reconfiguration statement. That is a genuinely earlier negative result in the neighborhood, but a different theorem: its statement and proof do not contain the present rank-four, nine-element deletion-partition obstruction, and no reduction between the two results has been established in either direction. The review explicitly does not claim global non-subsumption; it found no complete reduction from [2] (or from the one-way implications of [1]) to this candidate, which is bounded evidence. Earlier structural papers in the citation neighborhood (Blasiak; Bonin; Lasoń-Michałek; regular-matroid reconfiguration) concern connectivity, exchange sequences, saturation, or restricted classes, and yield no printed instance of this failure.

8. Reproducibility

The supplementary archive (Table 8) contains, unchanged, every file of the version-1 source-only archive: the 12 Lean modules with their SHA-256 manifest, the machine-checkable certificate (all 76 bases, both D-lists, exchange flag), the ordinary verification script and its log, and the replay scripts. Version 2 adds the MathDB problem record, the review reports with their stated limits, the replay receipts, HTTP receipts (URL, time, status and SHA-256) for the primary sources, the manuscript generator, and a script checking the corrected worked instance. Third-party source documents are identified by receipt and are not redistributed. The source-only files rebuild the endpoint from user-supplied Lean binaries and package caches. Its original replay.sh is retained as a historical convenience script only: it inherits the ambient PATH, HOME and Lean environment, sets just LEAN_PATH, a pinned-binary PATH prefix and MALLOC_ARENA_MAX=2, and enforces no CPU affinity, no CPU/wall/RSS/address-space limits, no nonempty-build-dir rejection and no dependency-hash verification.

mathdb:2611.00061v2 17 Nov 2026

The enforced replay path is capped_replay.py (Python standard library): safe archive extraction with per-member SHA-256, a fresh build directory with no preexisting .olean, a minimal private HOME, a pinned-binary PATH, fresh LEAN_PATH over the external packages, lean -j1 -M3500 -DwarningAsError=true and leanchecker per module, each call under ≤2-CPU affinity, CPU 120 s, wall 140 s, a sampled-RSS 6 GiB guard and a 12 GiB address-space rlimit (64 GiB hard guard), with every command, environment, exit status and resource peak logged. No prebuilt .olean files or large caches are shipped, and the external cache is not treated as a source-to-olean attestation. Every number in Sections 2-5 was rederived from the certificate; the recheck log is in the supplementary archive.

Appendix A. The complete circuit list

For completeness and independent checking we record all 23 circuits of M, sorted by size. A reader can verify each entry against Table 2: a circuit is dependent, while all its proper subsets extend to bases of Table 2.

{1, 4}	{0, 6, 8}	{2, 6, 7}
{3, 5, 6}	{0, 1, 2, 3}	{0, 1, 5, 7}
{0, 2, 3, 4}	{0, 2, 7, 8}	{0, 3, 5, 8}
{0, 4, 5, 7}	{1, 2, 5, 8}	{1, 3, 7, 8}
{2, 3, 5, 7}	{2, 4, 5, 8}	{3, 4, 7, 8}
{0, 1, 2, 5, 6}	{0, 1, 3, 6, 7}	{0, 2, 4, 5, 6}
{0, 3, 4, 6, 7}	{1, 2, 3, 6, 8}	{1, 5, 6, 7, 8}
{2, 3, 4, 6, 8}	{4, 5, 6, 7, 8}	

Table 7. All circuits of M; the two-element circuit {1,4} is the only nontrivial parallel class.

source-only/ (20 files)	the version-1 source archive, byte-identical: src/*.lean (12 modules), lean-source-manifest.json, matroid-certificate.json, ordinary_check.py and its log, historical replay.sh, enforced capped_replay.py
records/332371.jsonl	the MathDB problem record as retrieved (statement, source and status fields)
records/source-receipts/	URL, UTC timestamp, HTTP status and SHA-256 for arXiv v1-v5, the journal page, the workshop booklet [3] and [2]; source files not redistributed
reviews/	the independent literature/mathematics review, the Lean-fidelity review and the module-origin audit supplement, with their stated limits
verification/	per-stage lean and leanchecker receipts, negative controls and the replay summary
paper-src/, check_correction.py	the generator of this PDF and an independent check of the corrected worked instance

Table 8. Map of the supplementary archive (full inventory and SHA-256 hashes in its MANIFEST.sha256).

References

[1] M. Lasoń, *On the toric ideals of matroids of a fixed rank*, arXiv:1601.08199 (v1-v5, 2016; Conjecture 13 in v2/v3, Conjecture 2.5 in v4/v5); journal version: Selecta Mathematica 27, no. 2, article 18 (2021), DOI 10.1007/s00029-021-00633-6. HTTP receipts for all versions and the journal page are in the supplementary archive.

[2] M. Larson, *Counterexamples to two conjectures about matroids*, arXiv:2607.02208 (July 2026), Example 1.4 and Theorem 1.5.

[3] *Order and Geometry Workshop 2016*, problem booklet, p. 2 (independent restatement of the same conjecture); HTTP receipt in the supplementary archive.

[4] MathDB entry 332371: *Common-basis conjecture for partitions of a matroid ground set*; the retrieved record is in the supplementary archive.

[5] Lean 4.34.1 and mathlib d13f23b723b8a846827a245b89c10fc7d3f11612; pinned dependency manifest and replay scripts in the supplementary archive.

mathdb:2611.00061v2 17 Nov 2026