A chain-free pure YY-Delta minimum that is not minimal

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Abstract. We give an explicit doubled simple cubic graph for which exhaustive pure YY-Delta reduction attains a chain-free graph that is not minimal under the two degree-preserving moves. The input has 18 vertices. Its labelled pure reduction DAG has 2328 states and 9846 legal-move arcs; its smallest order is 11, attained by 25 labelled matrices. A selected minimum admits the 7-move sequence below to order 10. Thus Conjecture 5.1 of Jeffries and Yeats [1] is false under the illustrated, edge-distinct reading of its chain definition. This qualification is part of the statement, not a claim that the verbal definition is unambiguous in isolation.

The Lean endpoint is Round2.literal_negation : not Literal.Conjecture51. We reuse the previously replayed packet byte-identically; this manuscript task does not run Lean. The finite result has modest significance: it disproves the proposed criterion for doubled cubic inputs, without providing a replacement criterion or a new general simplification theory. No priority claim is made. This draft and its computation scripts were prepared with OpenAI Codex assistance and require root review; accepted credit remains zero.

1. The source problem and its context

Jeffries and Yeats study a degree-preserving variant of delta-wye transformations, motivated in part by graphs and Feynman periods. We use the journal version of [1], Definition 2.1 and Conjecture 5.1, with the arXiv version retained for comparison. The journal distinguishes two notions: a minimal graph has the least vertex count in its equivalence class; a minimum graph additionally has the least number of connections among minimal graphs. The conjecture first takes a smallest pure-reduction result, which we call a pure minimum. This does not assert either equivalence-class minimality or connection minimality.

Conjecture 5.1 (verbatim, journal p. 678; typography normalized):

Given a doubled 3-regular graph G, perform the YY-Delta transformation on all wyes in all possible ways to find a graph G' of the smallest size. Then G' is a minimal graph if and only if it does not contain a chain of one or more 3-cycles, each connected by a vertex, which is also doubly adjacent to one more vertex, where the 3-cycles on either end of the chain have one vertex with exactly 4 neighbours and no connections of size greater than 2.

The surrounding remarks are essential. The source allows the depicted vertices to coincide but requires distinct edges. It says other Figure 14 gadgets already certify nonminimality of arbitrary graphs using an expansion first. The conjectured extra assertion concerns pure minima from doubled cubic inputs. Its examples support a heuristic, not a proved classification. Our example violates exactly that extra assertion.

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2. Moves and the illustrated chain predicate

All graphs here are finite undirected loopless multigraphs. A symmetric nonnegative integer matrix A with zero diagonal records edge multiplicities; the degree is its row sum. A connection is the entire nonempty set of edges between a pair of vertices. Doubly adjacent means that the ambient connection has size exactly two. A doubled graph replaces each edge of a simple graph by two parallel edges.

Definition 2.1 removes a degree-six centre x with exactly three distinct neighbours u, v, w, each joined to x by two edges. It deletes x and its incident edges, and adds one edge on each of uv, uw, vw. This is YY-Delta (reduction). Its inverse, Delta-YY (expansion), selects distinct u, v, w with a positive entry on each triangle pair, subtracts one on those pairs, and adds a new centre doubly joined to each. Existing parallel edges on the triangle pairs are allowed. Each move preserves degree six. Equivalence means a finite sequence of either kind, including the empty sequence; pure reachability uses reductions only.

For precision, the chain predicate follows the verbal description together with Figure 14(a), (b), (d). Take distinct base vertices b[0],...,b[t] for a positive number t of triangles. Triangle i uses b[i], b[i+1], and an apex a[i]. Each end base has two double attachments; each internal base has one double attachment. End bases have four actual neighbours and no ambient connection larger than two. Every base has degree six. Apex and attachment vertices may coincide with one another or other vertices whenever the edges can remain distinct.

For every unordered pair, count all schematic edge occurrences, with a double attachment contributing two. Require that count to be at most the actual multiplicity A[u,v]. This capacity condition is precisely the availability of distinct parallel-edge copies in each pair’s finite edge pool. It forbids loops automatically. The bases must be distinct: otherwise their disjoint six incident edge ends would exceed a degree-six vertex’s capacity (and adjacent identified bases demand a loop).

A subtlety matters. A depicted internal double attachment reserves two edges, but under allowed vertex coincidences it can lie in an ambient triple connection, sharing the pair with a separate triangle edge. Imposing exact ambient double adjacency on every schematic attachment would exclude the review’s required positive alias control. Likewise, imposing internal row type [1,1,1,1,2] would be too restrictive. This paper uses the illustrated capacity reading fixed in the ruling and independently reviewed with that explicit qualification.

The earlier discovery lane used a restricted incidence diagnostic and incorrectly said no chain could leave an endpoint. That diagnostic is not our chain proof. The argument below allows first triangles through a triple connection and blocks them by residual capacities. The formal statement uses this same capacity predicate directly; no separate formal equivalence theorem to an edge-ID embedding API is claimed.

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3. Explicit input and a global pure minimum

Let C be the simple graph on vertices indexed from zero whose complete edge list is:

```
(0,3)  (0,6)  (0,14)  (1,2)  (1,9)  (1,16)
(2,5)  (2,16)  (3,7)  (3,14)  (4,6)  (4,12)
(4,15)  (5,9)  (5,13)  (6,11)  (7,8)  (7,11)
(8,10)  (8,17)  (9,12)  (10,11)  (10,14)  (12,15)
(13,16)  (13,17)  (15,17)
```

Every vertex has exactly three distinct neighbours, as checked directly from this list. Let G be its doubled graph, with matrix entries two on these pairs and zero otherwise. Thus G satisfies the doubled-cubic hypothesis without auxiliary assumptions on arbitrary inputs.

Apply pure reductions at the following CURRENT indices, deleting a vertex and closing the gap in the numbering each time:

```
reduce at 0, 0, 3, 4, 6, 7, 11
orders 18 -> 17 -> 16 -> 15 -> 14 -> 13 -> 12 -> 11
```

The resulting H = G’ has the following full adjacency matrix. The row and column indices run from zero in their displayed order.

```
0 0 0 0 0 2 0 1 0 0 3
0 0 0 1 1 0 1 0 3 0 0
0 0 0 2 0 1 0 0 0 3 0
0 1 2 0 0 0 2 0 1 0 0
0 1 0 0 0 0 2 1 1 1 0
2 0 1 0 0 0 0 1 0 1 1
0 1 0 2 2 0 0 0 1 0 0
1 0 0 0 1 1 0 0 0 1 2
0 3 0 1 1 0 1 0 0 0 0
0 0 3 0 1 1 0 1 0 0 0
3 0 0 0 0 1 0 2 0 0 0
```

The complete pure-reduction DAG uses labelled matrices and increasing survivor numbering, without an isomorphism quotient. Its distribution by order is:

```
Order:     11    12    13    14    15    16    17    18
States:    25   258   688   777   435   126    18     1
```

There are 2328 states and 9846 legal-move arcs, counting distinct legal centres even when their successor matrices coincide. A fresh checker starts from the edge list, tests every row for exactly three nonzero entries all equal to two, and recursively visits every resulting matrix. It also checks the supplied DAG root, distinctness of entries, exact successor sets and every listed child matrix. The order decreases on every arc, so exhaustion terminates.

Closure gives a proof rather than a heuristic: induction on any pure path keeps its current matrix in the finite closed set. Every entry has order at least 11, and the displayed path attains that order. Arbitrary survivor numbering gives an isomorphic graph; the literal formal proof separately transports such paths to canonical numbering. Consequently H is a globally smallest pure result, not merely an irreducible terminal graph. All 25 labelled results of smallest order are chain-free under the same exhaustive capacity search; only the selected H is needed for refutation.

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4. Chain absence by local edge capacities

In H the only possible end bases are 3, 6. Each has single neighbours 1 and 8; their mutual connection has size three. The first triangle of a chain starting at either end must therefore use that end together with 1 and 8. This conclusion uses the actual end row [1,1,2,2] and both prescribed end attachments: its double connections are already consumed, leaving only its two single edges for the triangle.

Orient the first triangle so the next base is 1; the case with next base 8 is identical by inspecting the displayed rows. It cannot be the other end of a one-triangle chain, since neither 1 nor 8 has the required end row. Its internal attachment must use the two remaining edges of the triple connection between 1 and 8. All other available connections at that base are single.

If the initial end was 6, the remaining candidates for the next triangle are 3 and 4. There is no edge between 3 and 4, so no second triangle fits. If the initial end was 3, the candidates are 4 and 6, joined by a double connection. Either orientation of this second triangle fails to continue: choosing 4 as the next base leaves no available double attachment at 4. Choosing 6 also leaves no available double attachment there, because its double connection to 3 was already consumed as an attachment at the starting end. Termination at 6 would require that same consumed connection, so it fails as well. Vertex 4 cannot terminate because it is not an end base.

These cases exhaust all first and second triangles, including allowed apex and attachment coincidences. No longer chain survives its initial prefix. Thus H has no illustrated edge-distinct chain. The fresh capacity search independently confirms the result and also checks every labelled pure minimum. The local proof is the reason, not a replacement of the predicate by a row-type heuristic.

5. A legal descent in the full equivalence class

Starting at H, perform the following operations, always using current indices. An expansion appends its new vertex; a reduction closes the deleted index gap.

```
1. expand [1, 6, 8]; order 11 -> 12
2. reduce 6; order 12 -> 11
3. expand [4, 7, 10]; order 11 -> 12
4. expand [3, 7, 10]; order 12 -> 13
5. reduce 10; order 13 -> 12
6. reduce 7; order 12 -> 11
7. reduce 9; order 11 -> 10
```

Each expansion uses a genuine triangle; each reduction removes a row consisting of three double connections. The fresh checker verifies symmetry, zero diagonal, nonnegative multiplicities and degree six after every operation; all intermediate matrices are in recomputation.json. The final order is 10, strictly below 11. Hence H is not minimal in its equivalence class. We do not claim that this final graph is globally minimal.

Combining pure minimality, chain absence, and this strict descent contradicts the chain-free implication of the conjectured biconditional for this doubled cubic input. Other pure minima are nonminimal too: reverse their path to G, follow the displayed path to H, and then take the descent.

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6. What Figure 14 does and does not explain

No printed Figure 14 gadget occurs in H under the checked edge-distinct interpretation: the chain argument excludes the first-column cases, and direct capacity embeddings exclude (c), (e), (f), and (g). This is not a classification of the source's entire infinite gadget family.

The shrinking trace nevertheless exposes source-known gadgets. After expanding (1,6,8) in the shorter witness, Figure 14(c) embeds with its two shared triangle bases at 1 and 8, its other triangle vertices at 4 and 3, and their shared double-attachment image at the new vertex 11. The two remaining attachments at base 3 go to 2 and 6. Distinct schematic vertices can share that new attachment image because the edge pools are separate.

The original discovery trace contains a commuting inverse detour: first expand (0,5,7), then expand (1,6,8), reduce 6, remove the first newly introduced vertex at current index 10, and continue with the last five moves of the shorter witness. After its two expansions, both (c) and (g) embed, with full mappings in recomputation.json. Removing the detour leaves the shorter descent. Thus (c) is exposed after one expansion in the shorter trace; both (c) and (g) occur after two expansions in the original trace. We do not claim (g) occurs after the shorter trace's first expansion: the fresh template check finds none there.

This timing distinction sharpens the brief's shorthand rather than changing the counterexample. The source already knew these gadgets reduce arbitrary graphs and knew the usefulness of expansion-first operations. The new finite obstruction is their hiding behind pure reduction from a doubled cubic input whose smallest pure results lack chains. It is not a novel local move or a new proof that the printed gadgets shrink graphs.

7. Literal formal correspondence and certificate

The source-only Lean packet is retained as lean-source.zip with SHA-256:

`c0dba5dc4f6738531b09750bd5e6f46ba565ba952f50fbe927e4f9cfe4a0e08c`

Literal.Graph is a finite symmetric loopless natural-number matrix with no global multiplicity bound. YYDelta implements the exact centre row and allows arbitrary bijective survivor numbering; DeltaYY is its inverse. PureMinimum quantifies over all pure results, whereas Minimal quantifies over the equivalence class generated by both moves. DoubledCubic uses a simple cubic adjacency matrix multiplied by two. Literal.Chain uses the capacity presentation above. Literal.Conjecture51 universally quantifies over doubled cubic inputs and all their pure minima.

The 634-module packet contains 582 closure chunks, each handling four fixed matrices. RawClosure.indexed_extend proves generic certificate soundness using exact graph equality and child indices, not external hashes. RawResult.canonical_original_lower supplies the finite lower bound. Transport.canonical_transport covers arbitrary labels. ChainPrefix.no_chain and the chain-absence modules cover the literal capacity predicate. Round2.literal_negation combines these with the explicit input and descent, without a remaining mathematical premise.

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8. Audits, replay history, and trust limits

The earlier rounds must not be presented as completed replays. Round one left the canonical certificate, labelling transport and literal chain absence unfinished. Round two proved the latter bridges, but its own certificate replay was partial. Independent fidelity review passed the definitions and bridges with the chain-reading qualification; its full-replay gate failed at Closure276 under the unchanged internal memory cap. Thus that review's overall recommendation was HOLD, not acceptance clearance.

Round three split combined arc-census computations into per-node kernel checks followed by small sum proofs. It also replaced broad tactic imports with narrower imports. The recorded reconstruction check says graph matrices, child lists, lower-bound proofs and original theorem statements were unchanged. Resource limits were not raised. The census count is ancillary to the lower bound, but remains proved rather than discarded.

Round three records a fresh serialized complete replay, and the root subsequently records its own fresh replay of every packet module with warning-as-error and explicit-module leanchecker. The root's replay-result.json reports PASS, a clean source scan and a defining-module metadata audit excluding unsafe, partial and substituted implementations. Its printed endpoint dependencies are only propext, Classical.choice and Quot.sound. These are inherited, hash-bound receipts, not a fresh Lean execution by this manuscript worker.

The safe-extraction test here checks archive paths, duplicate names, symlinks, containment, member bytes, and counts 634 Lean sources including 582 closure chunks. It executes none of them. The packet is copied byte-identically. The new Python checker reconstructs the graph mathematics independently from source definitions; it is useful corroboration, not a substitute for kernel checking.

The trust base includes the stated interpretation of the source, the Lean kernel and ordinary foundational axioms, pinned imported dependencies, and integrity of the replay environment and receipts. No full upstream dependency rebuild is claimed. The capacity predicate directly formalizes edge-disjoint occurrence; the packet does not prove equivalence to an independently implemented edge-ID API. Rendering and numerical checks cannot settle a competing reading of ambiguous source prose.

9. Related work, limits, and disposition

The primary mathematical antecedent is [1], including its known non-chain gadgets and expansion-first strategy. The earlier independent review inspected source sections on minimality and simple graphs and found no source-provided doubled-input pure-minimum counterexample. Its bounded literature checks included author listings, citing works, and the primary Panzer-Yeats discussion of doubled cubic graphs and Martin invariants [2]. These are reported searches with stated coverage limits, not a worldwide novelty theorem. Other papers were title-screened or abstract-screened rather than all read in full.

The root's later registry-refresh.json records CLEAN for complete-tree path screening of the then-current OpenAI and Omega registries. Path screening is not theorem-body screening. This task makes no new network query and makes no present-day exhaustive prior-art clearance. MathDB's archived open label and empty solutions list likewise establish neither novelty nor acceptance.

The result is a small computational refutation of one proposed criterion, under an explicit illustrated reading. It supplies no replacement characterization, no minimality algorithm, no general period identity, and no assertion about the neighbouring non-bipartite lower-bound conjecture. Significance is modest and qualified as ruled by root. No publication, upload, or acceptance is performed here.

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10. Reproducible dossier and references

The dossier separates mathematical data from inherited replay evidence. PROOF.md gives the argument and exact hypotheses. recomputation.json gives the freshly generated matrices, counts, operations and gadget mappings. scripts/recompute.py implements the source moves without importing the discovery or review checkers. sources/ preserves the exact MathDB raw record, cached source PDFs and text, the certificate, reviews, formal round documentation and relevant receipts. inputs-before.json and inputs-after.json bind the frozen input selection; the copy receipt gives original paths and hashes. A supplemental manifest covers included provenance outside that initial selection.

To repeat the numerical check without Lean, copy the dossier to a new permitted directory, retain the original sibling inputs or follow REPRODUCE.md's source-path arrangement, then invoke the capped checker with python3 -I. All work stays local. The checked extraction is only a source inspection area; reruns must use a fresh extraction target. No packet script is executed during this manuscript task. The supplied script's strict assertions fail if the certificate root, matrices, closure, minima, chain search or witness disagrees.

The full-dossier ZIP includes the editable manuscript, PDF, byte-identical Lean packet, safe-extraction manifest, fresh recomputation and visual QA receipt. The standalone packet's identity is also recorded separately. The ZIP's own hash belongs to an external receipt to avoid a circular self-hash; the internal deliverable manifest excludes itself and the archive. Root alone can accept the candidate. This implementation contributes zero accepted credit.

References

[1] Shannon Jeffries and Karen Yeats. A degree preserving delta wye transformation with applications to 6-regular graphs and Feynman periods. Annales de l'Institut Henri Poincare D 11 (2024), 641-682. DOI: 10.4171/AIHPD/172. arXiv:2110.07764v2. Bibliographic fields are verified in the retained journal source and review receipts. Definition 2.1, Figure 14, and Conjecture 5.1 are the controlling passages.

[2] Erik Panzer and Karen Yeats. Feynman symmetries of the Martin and c2 invariants of regular graphs. arXiv:2304.05299. The earlier independent review retained the primary text and inspected its section on doubled cubic graphs. No DOI is asserted here.

[3] MathDB record 352253. Subgraph criterion for minimal doubled 3-regular graphs. The exact archived JSONL line is included in sources/352253.raw.jsonl, with SHA-256 in receipts/source-copies.json. Its archived status is provenance, not a claim about current public status.

Internal evidence (not publications): delta-review-r1, independent mathematical review and recomputation; delta-fidelity-review-r1, qualified definition/bridge review with round-two replay failure; delta-formal-r1 through r3, implementation and change history; delta-parent-final-r2, root replay PASS and bounded registry screen CLEAN; delta-root-ruling-r1.json, qualified modest-significance ruling. All are identified by paths and hashes in the dossier rather than represented as external peer review.

Draft preparation note. This text, scripts, and dossier assembly were produced with AI assistance. There are no invented human authors, affiliations, or endorsements. Mathematical acceptance and any future authorship, dissemination, or priority assessment remain outside this implementation task.

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