# Gcd symmetry of recurrent-component sizes on a bidirected cycle

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## Abstract

For the sinkless rotor network on the bidirected cycle, Chan and Levine conjectured that the number of recurrent configurations of positive level m and weight i depends only on gcd(n,m,i). We prove the assertion for every n >= 3 and m >= 1. After identifying recurrent configurations with a finite cycle-test count, a weighted matching expansion expresses its generating function as a two-state transfer trace. At a primitive e-th root of unity, the one-period trace collapses to 2 + q_e t^e, where q_e is 0 for odd e and 2(-1)^(e/2) for even e. Consequently the Fourier coefficients depend only on the frequency order. Rotation and multiplication by units then give exactly the required gcd orbits. We also justify existence and uniqueness of the components. A frozen 75-module Lean source packet contains the unconditional network-language endpoint. Its reported checks, independent fidelity review, and remaining root verification are distinguished below.

## 1. Source network and exact question

Let V = Z/nZ, with n >= 3. The bidirected cycle has edges from k to k-1 and from k to k+1. A configuration is (x,q), where x:V -> Z records chips and q(k) is one of the two outgoing edges at k. As in Example 3.11 of [CL], a legal firing at k requires x(k)>0; first advance the rotor to the other edge, then move one chip along that new edge. Thus a current right rotor fires left and becomes left. A current left rotor fires right and becomes right. A legal execution is a finite sequence of such firings, possibly empty. Write a ->* b for reachability.

Following Definitions 4.6 and 4.7, two configurations are related when they have a common legal descendant. This is an equivalence relation by the exchange lemma; its equivalence classes are the legal components. They are not configurations modulo spatial rotation. Definition 5.2 calls a configuration a recurrent if it admits a nonempty legal execution and every configuration related to a can legally reach a. In particular recurrent configurations in one component are mutually reachable.

Definition 4.8 calls a component recurrent if it contains a diverse infinite legal walk: every vertex fires infinitely often. Forward, backward, and two-sided walks are allowed. These definitions distinguish a recurrent configuration from its whole legal component, which can also contain transient configurations. Source Lemma 5.19 identifies recurrent components with components containing recurrent configurations under the finite, locally irreducible, strongly connected, critical hypotheses satisfied here.

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## 2. Level, weight, and the main theorem

The network level is the total chip count L(x,q)=sum_k x(k). Indeed the source level formula is cap(N)-cap(q)+s^T x (Definition 5.17); for an agent network cap(N)=0 and s=1, while Lemma 5.18(iii) gives cap(q)=0. The rotor network is an agent network and is irreducible ([CL], Section 3.6, not Section 3.5).

Assign weight 1 to a left edge and 0 to a right edge. The configuration weight in V is

```
wt(x,q) = sum_k k x(k) + #{k : q(k) is left}  (mod n).
```

Both L and wt are invariant under firing. A move left decreases positional weight by 1 and increases rotor weight by 1; a move right does the reverse. The wrap around changes this calculation only by a multiple of n. Hence these invariants are constant on legal components. For m>=1 and i in V, write C_{n,m,i} for the unique recurrent component of level m and weight i, whose existence and uniqueness we prove in Section 3. Let r(C) denote the number of recurrent configurations in C.

Theorem 1 (network form; Conjecture 8.3 of [CL]). For every n>=3, m>=1 and i,j in Z/nZ, the components C_{n,m,i} exist uniquely, and

```
gcd(n,m,i) = gcd(n,m,j)  implies  r(C_{n,m,i}) = r(C_{n,m,j}).
```

Here uniqueness means uniqueness of the whole component, not of its recurrent representative. The gcd is independent of the integer representative of i because gcd(n,m) divides n.

For a natural-chip configuration write q(k)=+ for right and q(k)=- for left. A facing pair is an edge (k,k+1) with q(k)=+ and q(k+1)=-. Define R_{n,m} to be the set of configurations with x(k)>=0, sum x=m, and every facing pair containing a chip. In addition, when all rotors are constant their Hamiltonian cycle must contain a chip. At positive m this last condition is automatic. Set N_{n,m}(i)=#{a in R_{n,m}:wt(a)=i}.

Theorem 2 (count form). With these definitions, r(C_{n,m,i})=N_{n,m}(i), and N_{n,m} is constant on equal-gcd fibres. No division by n or quotient by rotation enters this definition. Finiteness follows because 0<=x(k)<=m and there are only 2^n rotor states.

The source's Table 8.1 gives r(C_{n,n,i}) for 3<=n<=8 and observes near equality of the entries. Rotational symmetry already explains equality when i-j is divisible by gcd(n,m); it does not by itself prove Theorem 1. The trace calculation below supplies the additional symmetry. A worked subset of that table, together with other levels, appears in Section 7.

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## 3. Recurrence, component uniqueness, and the reduction

We use the source cycle test (Theorem 7.4, equivalently its rotor special case Theorem 1.3): on a finite, locally irreducible, strongly connected agent network, recurrence is equivalent to nonnegative chips, local recurrence of q, and a chip on every directed rotor cycle. All rotor states here are locally recurrent by Section 3.6 of [CL]. The hypotheses hold for the bidirected cycle. Its directed rotor cycles are precisely facing pairs and, for constant rotors, the two Hamiltonian cycles. A directed walk that reverses direction immediately closes a two-cycle; without a reversal it traverses the entire cycle. This proves the asserted list, including n=3.

Thus the literal Definition 5.2 recurrence is exactly the test in Section 2. In particular all signed recurrent configurations are nonnegative. Their total level is positive, since a recurrent configuration has a legal first step. Their counts at fixed level are finite by the coordinate bound. The natural-chip model therefore neither loses nor duplicates any signed recurrent configuration.

For completeness, we prove uniqueness using stated source theorems, rather than citing the source's unproved phrase "one can check". Proposition 7.9 gives exactly |Tor(N)| recurrent components at each positive level. Its hypotheses are the finite, locally irreducible, strongly connected agent hypotheses already checked. By Theorem 5.25(i), and the rotor kernel K=2 Z^V and production matrix P=A/2 of Example 3.11,

```
Tor(N) = Z^V_0 / (2I-A) Z^V,
```

where Z^V_0 consists of vectors with coordinate sum zero. This quotient has order n, as the following elementary calculation verifies without a spanning-tree enumeration. Let e_k be the standard basis and let g_k be the class of e_k-e_0. The Laplacian relations say 2g_k-g_{k-1}-g_{k+1}=0, cyclically. Starting from g_0=0, these give g_k=k g_1, and the wrap relation gives n g_1=0. The map z -> sum k z(k) modulo n annihilates every Laplacian column and sends g_1 to 1. It follows that the quotient is exactly Z/nZ, not a smaller cyclic quotient. Hence there are exactly n recurrent components at level m.

All n weights occur. For i represented by 0<=i<n, put m chips at 0 and choose left rotors at positions 1 through i, with right rotors elsewhere. The weight is i. If i=0 the rotors are constant and their Hamiltonian cycle contains 0. Otherwise the only facing pair is (0,1), which also contains 0. The cycle test proves recurrence. Lemma 5.19 makes each corresponding component recurrent. Since distinct weights cannot share a component, these n examples exhaust the n components. Each weight therefore has exactly one recurrent component.

Every recurrent state of level m and weight i lies in that component, and every recurrent state in that component has those invariants. This proves r(C_{n,m,i})=N_{n,m}(i) and completes the semantic reduction. The Lean packet proves uniqueness by a different, direct sink-normalization argument, described in Section 8.

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## 4. Weighted matching expansion and transfer trace

Fix an n-th root z of unity. For m>=1 define W_m(z)=sum_{a in R_{n,m}} z^{wt(a)}, with W_0(z)=0. Exponents can use any integer lift of wt. Rotation of all chips and rotors by one vertex preserves the cycle test and adds m to the weight. Thus W_m(z)=z^m W_m(z). In particular W_m(z)=0 unless z^m=1.

Work with formal power series over C. Write g_k=1-t z^k and f_k=1/g_k. Temporarily impose only the facing-pair conditions, allowing every nonnegative chip total, and let G'(t) be the resulting weighted generating series. At level zero, a nonconstant rotor word has a right-to-left transition and hence an empty facing pair. Exactly the two constant rotor words survive; their weights are 0 and n, so each contributes 1. At positive level the facing-pair test is the complete cycle test. Therefore

```
G(t) := sum_{m>=0} W_m(z)t^m = G'(t)-2.
```

For a fixed rotor word, its facing pairs are disjoint: a vertex cannot be both the left and the right end of facing pairs. Inclusion-exclusion over facing pairs forced to be empty gives a sum indexed by matchings M of the n-cycle. For each selected edge (k,k+1), the rotors are forced to be (+,-); their rotor weight contributes z. Every uncovered vertex has two free rotor choices, whose weighted sum is 1+z, and a free chip series f_k. Consequently

```
G'(t) = sum_M (-z)^|M| (1+z)^(n-2|M|) product_{k outside V(M)} f_k.
```

The factor z per forced facing pair is essential. The sum includes the wrap edge (n-1,0) and both alternating perfect matchings when n is even. Multiply by D(t)=product_{k=0}^{n-1} g_k to obtain the polynomial

```
H(t) = sum_M (-z)^|M| (1+z)^(n-2|M|) product_{k in V(M)} g_k.
```

Define the ordered two-state matrices

```
X_k(t) = [[1+z, -z g_k], [g_k, 0]].
```

Then H(t)=Tr(X_0 X_1 ... X_{n-1}). To see this, expand the trace over all closed state words s_0,...,s_n=s_0 in {1,2}. A nonzero term has no consecutive state 2. Each transition 1->2 at k must be followed by 2->1 at k+1 and selects the matching edge (k,k+1), with weight -z g_k g_{k+1}. Every remaining transition is 1->1, with weight 1+z. Conversely each matching uniquely determines such a state word: state 2 occurs at the right endpoints of selected edges. This includes wrap excursions and the two perfect matchings. We have proved the generating identity

```
G(t) = Tr(product_{k=0}^{n-1} X_k(t)) / D(t) - 2.
```

All denominators have constant term 1, so every division here is legitimate in formal power series. The source's master determinant (Theorem 7.11) gives a related weighted generating function, but this derivation does not require it.

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## 5. The one-period trace collapse

Suppose z is primitive of order e, where e divides n, and put r=n/e. The matrices are e-periodic. With P=X_0 ... X_{e-1} the full product is P^r. The primitive-root factorization gives

```
D(t)=(1-t^e)^r,        det(P)=(1-t^e)^2.
```

Indeed product_{k<e}(1-t z^k)=1-t^e, and det(X_k)=z g_k^2, so the product determinant has the additional factor z^e=1.

Lemma 3 (trace collapse). The period trace is

```
Tr(P)=2+q_e t^e,
q_e=0 if e is odd;   q_e=2(-1)^(e/2) if e is even.
```

Proof. Write p(t)=Tr(P). Each matrix entry has degree at most one, so deg p<=e. Since X_k(zt)=X_{k+1}(t), with the indices read modulo e, cyclicity of trace gives p(zt)=p(t). Its coefficient p_j therefore satisfies (z^j-1)p_j=0. Primitivity implies p_j=0 for 0<j<e.

At t=0 all matrices equal B=[[1+z,-z],[1,0]], whose characteristic polynomial is (lambda-1)(lambda-z). Thus Tr(B^e)=1+z^e=2. This remains true when z=1: triangularization, or the Cayley-Hamilton trace recurrence, counts eigenvalues with multiplicity, with no diagonalizability assumption.

The coefficient of t in X_k is z^k J, where J=[[0,z],[-1,0]] and J^2=-z I. The top coefficient of p is therefore

```
p_e = z^(e(e-1)/2) Tr(J^e).
```

For odd e, J^e is a scalar multiple of J and has trace zero. For e=2s, the expression is 2(-1)^s z^(s(2s-1)+s)=2(-1)^s z^(2s^2)=2(-1)^s, since z^(2s)=1. This proves the formula also for e=1 and e=2. No asymptotic or numerical argument is used.

By Cayley-Hamilton, V_j=Tr(P^j) satisfies

```
V_0=2, V_1=2+q_e t^e,
V_j=(2+q_e t^e)V_{j-1}-(1-t^e)^2 V_{j-2}  (j>=2).
```

In particular V_r is an integer polynomial in t^e determined solely by e and r. Section 4 now yields

```
sum_{m>=0} W_m(z)t^m = V_r(t)/(1-t^e)^r - 2.
```

Hence W_m(z) is an integer depending on z only through its order e. Denote this integer by w_{n,m}(e). It vanishes unless e divides m. This order-only property, supplied by the trace collapse, is the extra symmetry needed beyond rotation.

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## 6. Fourier inversion and the arithmetic orbit

Finite Fourier inversion gives, for all residues i,

```
N_{n,m}(i) = (1/n) sum_{z^n=1} z^(-i) W_m(z).
```

This follows by expanding W_m and using sum_{z^n=1} z^(a-i)=n when a=i in Z/nZ and zero otherwise. Order-only dependence implies invariance under multiplication of i by any unit u modulo n: substitute z -> z^u in this sum. It permutes n-th roots and preserves their orders. Rotation separately gives N(i+m)=N(i). These two invariances agree with the route used in Lean.

We give the full arithmetic orbit argument. Put d=gcd(n,m). The subgroup m(Z/nZ) consists precisely of residues divisible by d, by Bezout's identity. Thus shifts by m connect exactly residues equal modulo d. If gcd(d,i)=gcd(d,j)=g, there is a unit v modulo d with vi=j modulo d. To prove this assertion, work prime by prime. For p^a dividing d exactly, equal truncated p-valuations mean either i=j=0 modulo p^a, when take v=1, or i=p^b I and j=p^b J with b<a and I,J units modulo p^(a-b). Choose v=J/I modulo p^(a-b) and lift to a unit modulo p^a. The Chinese remainder theorem combines these choices.

Any unit v modulo d lifts to a unit u modulo n. Start with an integer representative of v and require u=v modulo d. At primes dividing n but not d choose u=1 modulo their full prime powers; these conditions are compatible by the Chinese remainder theorem. At primes dividing d any lift of v remains nonzero modulo that prime. The resulting u is coprime to n. Thus j-ui is divisible by d and is b m in Z/nZ for some integer b, which may be replaced by a nonnegative representative. Hence

```
j = u i + b m  in Z/nZ.
```

Both operations preserve gcd(n,m,i); conversely the argument just given connects every pair with the same gcd. The count is therefore constant precisely on the required arithmetic orbits (without asserting that different orbits always have different counts). This proves Theorems 1 and 2.

An explicit alternative expression follows by grouping roots by order. Define c_e(i)=sum_{z primitive of order e} z^(-i), the Ramanujan sum. Then

```
r(C_{n,m,i})=(1/n) sum_{e | gcd(n,m)} w_{n,m}(e)c_e(i).
```

The weights w are obtained by coefficient extraction from Section 5. The prime-power unit argument above, applied with d=e, shows c_e(i) depends only on gcd(e,i). For e|d we have gcd(e,i)=gcd(e,gcd(d,i)), giving another direct proof of the gcd conclusion. This Ramanujan-sum formula is an ordinary exposition of the result; the formal endpoint instead uses finite Fourier injectivity and the unit-plus-shift orbit. The two routes use the same order-only Fourier statement.

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## 7. Worked counts and relation to the source evidence

The entries below are counts for the residues 0,1,...,n-1 in that order. The rows with m=n reproduce the corresponding rows of Table 8.1 of [CL]. The other rows illustrate the positive-level count at smaller m.

| n | m | r(C_{n,m,i}), in increasing i |
|---|---|---|
| 3 | 1 | 6, 6, 6 |
| 3 | 3 | 26, 24, 24 |
| 4 | 1 | 8, 8, 8, 8 |
| 4 | 4 | 122, 120, 118, 120 |
| 5 | 5 | 642, 640, 640, 640, 640 |
| 6 | 6 | 3630, 3624, 3624, 3630, 3624, 3624 |

For example take n=m=3. At e=1 the recurrence gives V_3=2+12t-6t^2, so [t^3] V_3/(1-t)^3 = 20+72-18=74. At e=3 we have V_1=2 and w_{3,3}(3)=2. The Ramanujan sums are c_3(0)=2 and c_3(1)=c_3(2)=-1. The formula therefore gives (74+4)/3=26 and (74-2)/3=24. For n=m=4 the order-1 coefficient is 480, the order-2 coefficient is 0, and the order-4 coefficient is 4, giving 122,120,118,120. These variations are allowed: the conjecture promises equality only for equal gcds.

## 8. Formal correspondence and architecture

The frozen packet contains 75 Lean modules, including audit and control modules. The exact final declarations are ChanLevine.mathdb_338887 and ChanLevine.recurrent_count_depends_only_on_gcd in formal/ChanLevineFinal.lean. The former asserts: for n,m natural with [NeZero n], n>=3, m>=1 and equal gcds, (a) there is a unique set S of signed configurations admitting a recurrent natural representative of level m and weight i with S equal to its legal component; (b) the canonical component C m i is a RecurrentComponent; and (c) r n m i=r n m j. The corollary projects (c). The nonzero instance excludes no n>=3.

| Source notion | Lean representation and bridge |
|---|---|
| Signed chips and rotor processing | SignedConfig; Dynamics.target, fire, Step |
| Legal executions and components | Reach; Related (common descendant); component |
| Definition 5.2 recurrence | Recurrent; NatRecurrent is recurrence of a lift |
| Definition 4.8 diversity | RecurrentComponent; SemPeriodic and Sem3Coverage |
| Level and source left-edge weight | Dynamics.level, weight; chipLevel, chipWeight |
| Cycle test and canonical component | CycleTest; canonical; C m i |
| Finite recurrent count | r via Nat.card; Cardinality and Reduction |

SourceSemantics proves natural/signed firing compatibility. Sem3 proves a closed word covers every vertex, nonnegativity of recurrent signed states, and reversal via word removal. Sem4 and Sem5 prove cycle-test sufficiency by a maximal execution with a quota of two firings per vertex: an unfinished deficit would give a nonempty chip-free rotor-closed set, forbidden by the test. Two firings at every vertex restore the configuration. For uniqueness Sem5 uses finite acyclicity to stabilize at sink 0, arranges rotor 0 to point right, then classifies the remaining rotors as a left prefix. Weight fixes its length, producing the canonical representative. This direct formal proof replaces the source torsion-group route used in Section 3.

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## 9. Discharged interfaces, audits, and trust limits

The original Endpoint module accepted seven ordinary Prop arguments. They were proof obligations, not added axioms; the final assembly supplies all of them.

| Frozen interface | Discharge in the merged endpoint |
|---|---|
| CycleTestBridge | Sem5.cycleTestBridge (SemFinal4) |
| CanonicalTest | canonicalTest (SemCanonical, SemFinal) |
| ConnectedFibres | Sem5.connectedFibres (sink normalization) |
| RotationCount | rotationCount (CountRotation, CountFinal) |
| UnitCount | Round6.unitCount, supplied both Round7 identities |
| ArithmeticOrbit | arithmeticOrbit (CountArithmetic, CountFinal) |
| AmbientBridge | Sem3.ambientBridge_unconditional |

The assembly merges the r5b semantic packet with r7c and r7t counting packets: 74 inherited modules, 36 shared names with identical bytes and no conflicts, plus ChanLevineFinal. The counting chain passes from finite weighted fibres and inclusion-exclusion through r7c's MatchingCountConvolution and r7t's MatchingTraceExpansion, then the coefficient quotient recurrence. Rounds 3-5 provide period reduction, denominator factorization, degree/invariance and top-coefficient trace collapse; finite Fourier injectivity supplies UnitCount. Earlier frozen comments saying "partial" describe their historical interfaces, now discharged. They do not create residual assumptions.

The source-only archive is byte-identical to the assembly delivery, SHA-256:

`9e685235e4d744adf9fdd5cafaccf874c6ef6fbccff4684003133df75da34bb0`

The pinned compiler is Lean 4.34.1; mathlib revision is d13f23b723b8a846827a245b89c10fc7d3f11612. Assembly reports strict serial compilation and leanchecker for all 75 modules after fresh extraction. Its audit starts at all 631 local declarations in the final import closure and reaches 28,176 declarations, reading opaque bodies and constructor edges; an independent traversal checks closure equality. It reports only propext, Classical.choice, Quot.sound as axioms; no unsafe, partial or _unsafe_rec dependencies; no local extern/implemented_by metadata. It separately inventories 64 upstream executable metadata entries. These are not additional axioms, and the complete imported executable environment is not claimed metadata-free.

Three expected-failure controls test differing gcds, omission of positive level, and reversed rotor weight; two positive controls pass. They are interface sensitivity checks, not proofs that every altered statement is false. The independent fidelity review freshly compiled and checked the 54-module endpoint import closure and examples, and separately compared legal-dynamics SCC counts with the cycle test and formula in 16 cases, plus four signed-state checks. Assembly and fidelity reports are preserved, not rerun in this manuscript lane. No Lean, lake or leanchecker was executed here.

The root replay receipt dated 3 November 2026, 16:08 UTC reports PASS: all 75 modules freshly compiled and checked, both endpoint axiom lists restricted to the three allowed axioms, the 28,176-declaration audit passed, and all controls passed. This is root's preserved report, not a replay performed in this manuscript lane. The proof still relies on Lean's kernel, the three listed axioms, trusted pinned upstream mathematics, source correspondence, and the integrity of the preserved receipts. Numerical agreement is diagnostic evidence, not the proof. The manuscript lane safely extracted and hash-checked the source packet, checked worked counts independently, and rendered and visually reviewed all pages. Root alone reads and accepts the result; this draft assigns zero credit.

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## 10. Related work, access limits, and reproducible evidence

The cycle test, general component theory, torsion-group classification, and master determinant are due to [CL] and the earlier literature they cite. The source credits its rotor recurrence theorem to Tothmeresz. We use [CL]'s stated theorems with their hypotheses rather than attribute a new general cycle test or component classification to this work. The special one-period trace collapse and its use to force order-only Fourier dependence are the central additions of the present argument.

The bounded adversarial review found no resolution of Conjecture 8.3 in its searches dated 3 November 2026. It checked cached arXiv v1/v2 bodies, the candidate proof, Chan's 2019 thesis, OpenAlex topic and author results, and the accessible AMS abstract page. It identified the source master determinant and classical transfer-matrix methods as known ingredients. The review judged significance a qualified pass: the proof is short and uses standard tools, so substantive credit remains a root decision. The root's earlier adjudication called the candidate eligible pending literal formalization, manuscript, and replay. Neither ruling is acceptance.

The published AMS 2022 full body was not freshly accessed. The v1 and v2 conjectures agree, but this does not prove every detail of the published body is unchanged. Citation coverage was sparse. Google Scholar, Crossref, Semantic Scholar and the arXiv search API were not checked by the review; the original cached registry investigation did not enumerate all commits between its recorded Omega heads. A later root refresh (16:06 UTC) screened complete current Omega D5/Problems trees at head 86172888e706177b6c1d135cb12aec3b6d2cdce8 and found no relevant path. OpenAI head 4eaee87548250b2682affda7d05d947c05554bae was unchanged; its returned recursive tree was truncated. This remains a bounded path screen, not a full-body search. No fresh network, account, browser, external message, publication or upload was used in this manuscript lane. Consequently we claim neither exhaustive novelty clearance nor worldwide priority.

The dossier preserves the exact MathDB JSONL record (including its historical "open" metadata), with SHA-256 928ea2c6a9491df0f45641d529dcbdee9fd75c3959890becfbcf04ae39077bd2. An open metadata label is not evidence of novelty. Cached arXiv texts, PDFs, access receipts and bounded search records are copied under evidence/ with input hashes. The source packet is included unchanged; the safe extraction is separately inventoried. PROOF.md contains the ordinary proof; REPRODUCE.md distinguishes document/count reproduction from the separately authorized root replay. Frozen-input hashes before and after, resource receipts, page images and QA receipts document this lane's operations.

The unique-existence predicate of the formal endpoint is deliberately recorded exactly: it concerns component sets with recurrent representatives. The source Lemma 5.19 identifies these with diverse-walk recurrent components. The packet proves the direction from recurrent configuration to diverse component needed by the endpoint, but does not package a standalone equivalence for every arbitrary set satisfying RecurrentComponent. We do not advertise that extra theorem as formalized. In the ordinary source-language statement, Lemma 5.19 justifies the full terminology.

## Bibliography

[CL] Swee Hong Chan and Lionel Levine. Abelian networks IV. Dynamics of nonhalting networks. Memoirs of the American Mathematical Society 276 (2022), no. 1358. DOI: 10.1090/memo/1358 (verified in the preserved independent review's OpenAlex/AMS record). arXiv:1804.03322, v1, 10 April 2018; v2, 14 November 2020. Source locations: Example 3.11; Definitions 4.6-4.8, 5.2, 5.17; Lemmas 5.18-5.19, 7.8; Theorems 5.25, 7.4, 7.11; Proposition 7.9; Section 8.3, Conjecture 8.3 and Table 8.1.

[Ch] Swee Hong Chan. Nonhalting abelian networks. Ph.D. dissertation, Cornell University, 2019. Cached thesis consulted in the preserved prior-art investigation; no DOI asserted.

[M] MathDB. Record 338887, "Conjecture on recurrent component cardinalities in sinkless rotor networks". Exact cached record and provenance preserved in this dossier; https://mathdb.com/p/338887/conjecture-on-recurrent-component-cardinalities-in-sinkless . The source theorem, not the record's open label, fixes the mathematical question.

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