A five-vertex counterexample to nearest-colour contact order

DRAFT - 3 November 2026 UTC. Prepared in the MathDB research campaign with AI assistance (OpenAI Codex). No human authorship or affiliation is asserted. Independent source-fidelity review, final replay and acceptance remain the responsibility of the campaign root. MathDB 365301; zero accepted credit.

Abstract

We exhibit a connected simple graph on five vertices whose two-colour Cauchy transform has contact order 6 at the boundary point corresponding to (infinity,0), although the root is adjacent to a w-coloured vertex. This contradicts Conjecture 6.1 of Tully-Doyle, Adlin, Thai and Tiscareno [1], which predicts twice the nearest-w distance. The two local branches have exact orders 6 and 2. We prove the result directly by the zero-set definition of contact, as well as by analytic level-curve expansions. The accompanying r6 Lean dossier proves the repaired local endpoint ContactPilot.refutationLocal, the literal negation of SourceConjectureLocal, including the matrix resolvent, polynomial reduction, stability and Cayley transfer. This draft makes no priority claim.

mathdb:2611.00033v1 04 Nov 2026

1. Source conventions and the statement

Let G be a finite simple undirected graph with adjacency matrix A. Colour each vertex z or w, let Y be the diagonal indicator of the z vertices, and choose a root i. The source representation is

```
M(z,w) = A - zY - w(I-Y),      f(z,w) = (M(z,w)^(-1))_(i,i).
```

It is the diagonal entry of the inverse, not its reciprocal. On the bi-upper half-plane, M is invertible: if Mv=0 with v nonzero, the imaginary part of v*Mv is strictly negative. With D=diag(Im z or Im w), the inverse identity is

```
Im(M^(-1)) = (M^(-1))* D M^(-1).
```

Thus Im f is positive. Real coefficients give real values on the real boundary wherever the quotient is defined. The Cayley map beta(v)=(1+iv)/(1-iv), with inverse i(1-v)/(1+v), gives a rational inner function

```
phi(Z,W) = beta(f(beta^(-1)(Z), beta^(-1)(W))).
```

The boundary point (infinity,0) becomes (-1,1). All contact orders below are local there. The source's Section 4, Theorem 4.6 proves contact order 2d when there is exactly one w vertex and its minimum path distance from the root is d. Conjecture 6.1 extends this assertion to multiple w vertices. The adjacent paragraph additionally predicts equal contact orders on all branches. Our example has two w vertices; it violates both expectations without contradicting Theorem 4.6.

For a reduced expression phi=q/p with p stable and q a unimodular multiple of its reflection, use the face |Z|=1, |W|<1. At a torus point (tau1,tau2) with q(tau1,tau2)=0, local contact K means the following exact quantifiers. First fix c>0, C>0 and rho0>0. There exists delta0>0 such that for every unit Z with 0<h=|Z-tau1|<delta0 and EVERY disk zero v of q(Z,v) with |v-tau2|<rho0, c h^K <= 1-|v|. Also, for EVERY 0<rho<=rho0 there exists delta>0 such that for every unit Z with 0<h<delta there EXISTS a disk zero v with |v-tau2|<rho and 1-|v|<=C h^K. The constants c,C,rho0 are fixed before both clauses; only the upper parameter interval may shrink with rho. In particular C cannot depend on rho, and an empty local zero set cannot supply the upper witness.

This compares the local zero-height infimum with h^K in arbitrarily small neighborhoods. Here (tau1,tau2)=(-1,1). Section 3.1 and Remark 3.2 of [2] retain the maximum branch convention in the ELA 2026 journal version: generic matched level curves yield the maximum of their vanishing orders. It is neither the minimum nor their sum.

2. The graph and the reduced resolvent

Use vertices 0,1,2,3,4, root 0, and the following data:

```
E = {01,02,03,13,24,34},    z vertices = {0,3,4},    w vertices = {1,2}.
```

The root has w neighbours 1 and 2. A path of length zero cannot reach a w vertex since the root is z. Consequently its nearest-w distance is exactly 1. The graph is connected: 1,2,3 are adjacent to 0, and 4 is adjacent to 2 (also to 3). In the indicated order its matrix is

```
M = [ -z   1   1   1   0 ]
    [  1  -w   0   1   0 ]
    [  1   0  -w   0   1 ]
    [  1   1   0  -z   1 ]
    [  0   0   1   1  -z ].
```

Expanding the determinant and the (0,0) cofactor gives f=N/Q, with

```
N = (wz-w-1)(wz+w-1),
Q = -w^2 z^3 + 2w^2 z + 4wz^2 + 2wz - w - 3z.
```

This is an identity of rational functions; the pointwise matrix-inverse identity is used where Q is nonzero. In the bi-upper half-plane that condition follows from the strict imaginary-part argument. The determinant and cofactor are independently recoverable from the displayed matrix, so this quotient is tied to the actual graph.

We verify reduction over C[z,w], not merely over the rational numbers. Each factor w(z-1)-1 and w(z+1)-1 is primitive as a polynomial in w over C[z]. It is linear and irreducible over C(z), hence irreducible by Gauss's lemma. They are nonassociate: their coefficients of w differ while their constant coefficients agree. Neither divides Q, since substitution yields

```
Q(z,1/(z-1)) = (2z-1)^2/(z-1)^2,
Q(z,1/(z+1)) = -1/(z+1)^2.
```

Both are nonzero rational functions. There are no z-only factors of N because its factors are primitive. Every common divisor of N and Q is therefore a unit. The formal proof obtains irreducibility using a polynomial equivalence and checks nondivisibility already at (z,w)=(0,-1) and (0,1). It concludes a common-divisor statement in the original bivariate complex polynomial ring.

The two distinct w factors matter: this is genuinely a degree-two representation in w after reduction. Removing a cancelled factor cannot turn it into the one-w case. Likewise, the distance calculation refers to the original graph, rather than an auxiliary quotient graph or an effective coupling parameter.

mathdb:2611.00033v1 04 Nov 2026

3. Analytic level curves and exact differences

For a finite nonzero real level L set t=1/z and w=tu. The cleared equation f=L is

```
F(t,u,L) = a_L(t)u^2 + b_L(t)u + c_L(t) = 0,
a_L = L(1-2t^2)+t-t^3,
b_L = -4L-2Lt+Lt^2-2t,       c_L = 3L+t.
```

Indeed F=t(N-LQ) after substitution. At t=0 it equals L(u-1)(u-3), with simple roots 1 and 3. The discriminant has value 4L^2, so a local analytic square root with initial value 2L exists. The quadratic formula gives two convergent analytic branches, uniquely labelled by their slopes. The leading coefficient is nonzero near zero, so these are all roots; in particular all w roots are O(t).

The identity N-LQ=0 away from t=0 is equivalent to f=L on these branches. If Q vanished there then N would vanish as well. The displayed substitutions in Section 2 show that common zeros cannot accumulate at z=infinity. Thus no level branch here is a spurious common-zero component.

The branches have expansions

```
w_(L,1)(t) = t-t^2+t^3-t^4/2-t^5
             +(17/4-1/(2L))t^6+O(t^7),
w_(L,3)(t) = 3t+(3-2/L)t^2+O(t^3).
```

These are convergent analytic expansions. For an explicit coefficient check for the longer jet, put

```
p_L(t) = 1-t+t^2-t^3/2-t^4+(17/4-1/(2L))t^5.
```

Substitution into F gives O(t^6). The identity F(t,u)-F(t,p)=(u-p)[a_L(t)(u+p)+b_L(t)] has bracket -2L at (t,u,p)=(0,1,1). Division by this analytic nonvanishing bracket shows u-p=O(t^6), proving the first expansion. For the slope-three branch, put p=3+(3-2/L)t; its residual is O(t^2) and the bracket has value 2L. This proves the second expansion without treating formal jets as analytic existence.

For distinct finite nonzero real L,H, subtraction gives

```
w_(L,1)-w_(H,1) = (1/(2H)-1/(2L))t^6+O(t^7),
w_(L,3)-w_(H,3) = (2/H-2/L)t^2+O(t^3).
```

The coefficients are nonzero; the orders are exactly 6 and 2. Cross-matching slopes gives order 1 and is not the local branch matching. The calculation works for every nonzero real level, not only positive levels. Consequently it is compatible with the generic-level convention of [2, Remark 3.2] and [3, Theorem 3.1]. The maximum order is 6. We next prove this conclusion directly from the zero-set definition, independently of the generic-level theorem.

mathdb:2611.00033v1 04 Nov 2026

mathdb:2611.00033v1 04 Nov 2026

4. A direct zero-set proof of contact six

For domain assertions use x=-1/z, which preserves the upper half-plane; t=1/z alone reverses it. Clearing the denominator of beta(f(-1/x,w)) gives

```
R(x,w) = x^3[Q(-1/x,w)-iN(-1/x,w)]
  = (1-ix-2x^2+ix^3)w^2
    +(4x-(2+2i)x^2-x^3)w+3x^2-ix^3.
```

Write A(x)=1-ix-2x^2+ix^3, B(x)=4-(2+2i)x-x^2 and C(x)=3-ix. For w=xu the equation is x^2[A(x)u^2+B(x)u+C(x)]=0. At zero the desingularized quadratic has roots -1,-3. Its normalized discriminant (B^2-4AC)/4 has value 1. The analytic inverse function theorem for s -> s^2 at 1 supplies a local analytic square root. The quadratic formula yields exhaustive analytic branches u_1,u_3. No global choice of square-root branch is required.

Here is an analytic remainder proof of their imaginary orders. Put

```
P(x) = -1-x-x^2-x^3/2+x^4.
```

Direct polynomial multiplication gives A P^2+B P+C=x^5 H(x), where H is polynomial with H(0)=-17/2+i. Subtract the branch equation; the divided difference A(u_1+P)+B has value 2. Therefore, on a neighbourhood,

```
u_1=P+x^5 g,       g=-H/[A(u_1+P)+B],
g analytic,        g(0)=17/4-i/2.
```

Similarly A(-3)^2+B(-3)+C=x H_3(x), with H_3(0)=6-4i. The divided difference A(u_3-3)+B has value -2. Thus u_3=-3+xh with h analytic and h(0)=3-2i. For real x the two pole roots w_j=xu_j satisfy

```
Im w_1(x) = x^6 Im g(x) = -x^6/2+O(x^7),
Im w_3(x) = x^2 Im h(x) = -2x^2+O(x^3).
```

The nonzero analytic factors give exact real analytic orders 6 and 2. After shrinking the interval, both roots have negative imaginary part. They tend uniformly to zero, and every root is one of them for nonzero sufficiently small x. Continuity gives |Im w_1| between |x|^6/4 and |x|^6, while |Im w_3| is at least |x|^2. Restricting |x|<1, every root has imaginary distance at least |x|^6/4, and the first root witnesses an upper bound |x|^6 in any prescribed small neighbourhood. The stronger formal attribution is half_plane_contact_six_localized in RootLocalization.lean: for every epsilon>0, one positive interval works for an all-root lower constant 1/4 and a witness of norm below epsilon with upper constant 1. The weaker HasHalfPlaneContact R 6 is HISTORICAL and is not a general faithful local definition.

There is no common component of R and its coefficientwise conjugate: one would divide the cleared N and Q, while x cannot divide R because R(0,w)=w^2. Thus the local boundary zero is not an artefact of common-factor cancellation.

5. Reduction on the bidisk and the Cayley comparison

For clarity the global certificate uses polynomial clearing rather than a merely local quotient. In C[Z,W], set a=i(1-Z), b=1+Z, c=i(1-W), e=1+W and define

```
N_c = b(ca-cb-be)(ca+cb-be),
Q_c = -c^2 a^3+2c^2 ab^2+4ca^2 be
      +2cab^2 e-cb^3 e-3ab^2 e^2,
q_b = Q_c+iN_c,       p_b = Q_c-iN_c.
```

Clearing gives N_c=b^3 e^2 N(a/b,c/e) and the analogous identity for Q_c. Hence phi=q_b/p_b in the bidisk. The proof of coprimality factors N_c into b and its two bilinear factors. Each is irreducible; exact evaluations at (1,-i), (1,i) and (-1,0) exclude their divisibility into Q_c. Invertible scalar linear combinations then give reduction of q_b,p_b. Thus reduction is checked after clearing, including its exceptional divisors; it is not inferred solely from rational coordinate substitution.

The support of p_b is bounded by (3,2). With reflection in that bidegree, direct polynomial coefficient computation gives q_b=reflect(p_b), with phase 1. The denominator is stable: beta^(-1) takes the bidisk into the upper half-plane, Q is nonzero there by invertibility of the graph matrix, and Q-iN cannot vanish because Im(N/Q)>0. Indeed Q-iN=0 would imply f=-i. The clearing factors are nonzero in the open bidisk. This gives a reduced rational-inner certificate for the actual graph function. Both q_b and p_b vanish at (-1,1).

For real nonzero x near zero let Z=-beta(x)=beta(-1/x). Coefficientwise conjugation of R converts lower-half-plane pole roots u to upper-half-plane roots conjugate(u), and W=beta(conjugate(u)) gives precisely the nearby numerator zeros. The clearing and quotient identities establish both directions; possible exceptional denominators are ruled out by localization. Every such image eventually lies inside the disk and arbitrarily close to 1.

The transfer uses explicit estimates, not an assertion that contact is conformally invariant. If Im y>0 and |y|<1/2, then

```
1-|beta(y)|^2 = 4 Im y / |1-iy|^2,
Im y/2 <= 1-|beta(y)| <= 16 Im y.
```

For |Z|=1 near -1 the circle chart x=i(1+Z)/(1-Z) is real and

```
|Z+1|/2 <= |x| <= |Z+1|.
```

Combining the half-plane lower constant 1/4, height lower factor 1/2, and parameter lower factor 1/2 in power six yields 1/512. The upper constant is 16. Localization first places all witnesses in any proposed numerator neighbourhood. Thus bidisk_local_contact_six proves HasLocalZeroSetContact (eval2 bidiskNumerator) (-1) 1 6: fix rho0=1/2, c=1/512 and C=16. One delta0>0 bounds ALL disk zeros within rho0. For EACH rho in (0,rho0], a possibly smaller delta>0 supplies a zero within rho with the SAME C=16. These constants are read from BidiskContact.lean and LocalBidiskContact.lean; the lower estimate is inherited, while the upper localization is proved separately.

Finally contact 2 is impossible, as formalized by local_contact_unique and bidisk_not_local_contact_two. Its positive quadratic lower bound, applied to the localized sixth-order witness, would imply c|Z+1|^2 <= 16|Z+1|^6 for arbitrarily small positive |Z+1|. Dividing and tending to zero contradicts c>0. The boundary point is therefore a genuine contact-six singularity, and 6 is unequal to twice the nearest-w distance 1.

mathdb:2611.00033v1 04 Nov 2026

6. Formal statement and correspondence

The Lean namespace is ContactPilot. SourceContactLocal.lean defines HasLocalZeroSetContact with exactly the fixed c,C,rho0, all-root lower bound and every-rho upper witnesses of Section 1. ReducedCertificate f stores the numerator, nonzero denominator, common-divisor reduction, stability, representation of beta composed with f composed with betaInv, bidegree bounds, unit phase and reflection identity. NearestWDistance requires an attained w-reaching walk of length d and minimality among all such walks.

SourceConjectureLocal quantifies every n, simple graph G on Fin n, Boolean coloring isZ, root i, certificate cert for graphFunction G isZ i, and natural d,K. Its implication is

```
NearestWDistance G isZ i d -> 0<K -> Even K ->
HasLocalZeroSetContact (eval2 cert.numerator) (-1) 1 K -> K=2*d.
```

The operative endpoint is ContactPilot.refutationLocal : not ContactPilot.SourceConjectureLocal. It applies the universal hypothesis to the five-vertex certificate, d=1 and K=6, using nearest_w_one and bidisk_local_contact_six, and contradicts 6=2. The graph is connected, has both colors and a z root. No residual branch or certificate hypothesis remains.

Correspondence table (declaration - meaning):

```
resolventMatrix, graphFunction - color-diagonal matrix, inverse entry
beta, betaInv                - half-plane / disk coordinates
ReducedCertificate, reflect  - reduced rational-inner representation
HasLocalZeroSetContact       - local zero-height comparison, Section 1
local_contact_unique         - uniqueness for arbitrary q and torus point
NearestWDistance             - attained shortest root-to-w walk length
SourceConjectureLocal        - universal nearest-w implication above
refutationLocal             - negation of that implication
```

The branch chain QuadraticBranches -> ExactOrders -> SecondBranch -> BranchBounds -> HalfPlane constructs analytic roots and exact orders. GraphResolvent ties N/Q to the matrix. Coprimality and CayleyPolynomials through BidiskCertificate supply the reduced certificate. RootLocalization, CayleyLocalization and CircleChart give localization and coordinate transfer. LocalBidiskContact proves bidisk_local_contact_six, reusing only the valid lower estimate from HISTORICAL bidisk_contact_six in BidiskContact.lean. ContactUniqueness proves local_contact_unique; RefutationLocal proves bidisk_not_local_contact_two and refutationLocal. The new upper proof uses half_plane_contact_six_localized, not a coercion from the old predicate.

mathdb:2611.00033v1 04 Nov 2026

6.1. Existence, uniqueness and the source bridge

For arbitrary functions q, arbitrary torus points and natural orders, local_contact_unique proves uniqueness. Combine the lower bound of one order and the upper witness of the other inside min(rho1,rho2), with a common sufficiently small parameter interval. If K1<K2, then c1 h^K1<=C2 h^K2 is impossible as h tends to zero through positive circle displacements. circle_punctured_small supplies those displacements at every unit point; the parameter domain is not vacuous. Swapping the orders gives equality. This theorem does not assert existence of any contact germ.

The ordinary existence bridge uses the source's LOCAL infimum/maximum-branch convention. Isolate the finite local numerator-root cluster at tau2. Its branch heights have positive power comparisons. Their maximum exponent K_src gives a lower bound for every branch, after restricting h<1, and an upper bound on a branch attaining that maximum. This branch tends to tau2, so it enters every smaller neighborhood with the same comparison constant. Roots outside the cluster are excluded by first shrinking rho0 and then delta0. Thus whenever the source order is defined in this sense, it satisfies HasLocalZeroSetContact. Conversely its two clauses compare the local infimum with h^K, and uniqueness prevents a different exponent. Graphs without such a germ are outside this explicit hypothesis; no invented order is assigned.

This is an ordinary statement-correspondence argument, not a general Lean analytic-contact existence theorem. Reduced representations of the same rational function differ by scalar units; enlarged reflection bidegrees allow monomial local units at (-1,1). These changes cannot create another local exponent. The input Cayley map sends (infinity,0) to (-1,1) independently of the graph; it does not assert a singularity for every graph.

For the actual counterexample, the exhaustive analytic branches in Sections 4-5 independently establish existence and K_src=6. The source conjecture would give K_src=2 for this graph, a contradiction. More generally the source conjecture implies SourceConjectureLocal by the preceding ordinary existence bridge and uniqueness. Calling an arbitrary contact hypothesis a restriction would not by itself justify that implication.

HISTORICAL: HasZeroSetContact, HasHalfPlaneContact and ContactPilot.refutation (negating the old SourceConjecture) belong to the r5 chain. Review r2 failed general locality: one existential radius could let an upper witness come from a different boundary cluster, even allowing orders 2 and 6 at the same named point. The root ruling agreed and held acceptance. The five-vertex mathematics survived because its roots actually localize. The r6 repair fixes the lower neighborhood and requires an upper witness in every smaller neighborhood with the same C. Review r3 passed A, B and bounded C, with D qualified pending manuscript integration. Historical declarations remain in the packet for provenance; they are not the operative local definition or endpoint.

mathdb:2611.00033v1 04 Nov 2026

6.2. Diagnostic locality regression (ordinary, non-Lean)

Let p6=p_b and q6=q_b be the pair of Section 5, and use the reviewer's diagnostic

```
p*(Z,W) = p6(Z,-W)(2+Z-W),
q*(Z,W) = q6(Z,-W)(2ZW+W-Z).
```

The preserved exact regression verifies reduction and bidegree-(4,3) reflection. Stability follows from stability of p6 and |Z-W|<2 in the open bidisk. At Z=-1 the first numerator factor is -8i(W+1)^2, nonzero on the compact closed ball |W-1|<=1/2. Uniform continuity excludes that factor's zeros there for Z sufficiently near -1. Every local zero therefore comes from the linear factor, with unique root v(Z)=Z/(2Z+1). It tends to 1 and thus enters every positive neighborhood of 1.

For |Z|=1 and h=|Z+1|>0, |2Z+1|^2=1+2h^2, so

```
1-|v(Z)| = 2h^2/[sqrt(1+2h^2)(sqrt(1+2h^2)+1)].
```

For h<1 this lies between h^2/3 and h^2, since the denominator is at least 2 and at most sqrt(3)(sqrt(3)+1)<6. The root lies in the disk for h>0. Take rho0=1/2, c=1/3 and C=1, and shrink delta0 for separation and h<1. The upper witness enters each rho in (0,rho0] after shrinking delta, preserving C=1. Hence the diagnostic has local order exactly 2. An alleged order-6 upper witness in the separation ball would force h^2/3<=C h^6 for arbitrarily small h, impossible. This also illustrates general uniqueness.

This diagnostic is an ordinary analytic argument with exact symbolic algebra, not a Lean theorem. No graph-resolvent realization of the product is asserted. The old predicate could borrow the remote order-6 cluster at -1 through a large radius; the repaired predicate cannot. See REGRESSION.md and the preserved r6 regression log for the full provenance.

mathdb:2611.00033v1 04 Nov 2026

7. Verification and trust limits

The repaired r6 source-only packet contains 33 Lean modules: 28 unchanged historical r5 modules and five additions, SourceContactLocal, ContactUniqueness, LocalBidiskContact, RefutationLocal and AuditR6. The delivered contact-source-only.zip is a byte-identical copy of contact-formal-r6/contact-r6-source.zip. Its SHA-256 is

`e92d6eceaf79972064c2f142a9cd1c4b3df45b520d1eed0445f288cddc1270d6`

The pin is Lean 4.34.1, commit 5045d0056413266e57c625dcd7c365b10e377c52, and mathlib d13f23b723b8a846827a245b89c10fc7d3f11612. The portable r6 driver refuses existing local compiled objects or a pre-existing build directory, compiles serially with -j1 and -DwarningAsError=true, invokes leanchecker separately per module, and inventories declaration types and opaque-inclusive bodies as expression DAGs. It rejects unsafe/partial declarations and transitive axioms beyond propext, Classical.choice and Quot.sound.

Supplied r6 evidence, preserved under evidence/r6/fresh-extraction/replay-logs, reports 33 strict builds, 33 checker calls, 323 declarations and 17 failing negative controls. complete.json binds the 33 source hashes; negative-controls-reviewed.json records the actual failures, including old-to-local shortcuts, wrong local order and reversed local refutation. The AuditR6 declaration DAG and audit inventory record allowed-axiom checks for refutationLocal, local_contact_unique and bidisk_local_contact_six. These are supplied r6 replay records, not computations run by this manuscript lane.

Review r3 independently checked the 26-module endpoint closure, one endpoint checker and three axiom prints. It explicitly did not run a full packet replay or all negative controls. Root's decisive fresh replay remains PENDING and is not claimed here. This lane ran no Lean or lake. The r6 driver caps each call at two CPUs, CPU 120 s, wall 140 s, sampled private memory 6 GiB and MALLOC_ARENA_MAX=2; manuscript generation uses the same CPU/wall ceilings and sampled total process-group RSS. REPRODUCE.md points to the r6 driver. The full dossier binds paper, sources, reviews, source receipts and replay records with SHA-256 hashes; no compiled objects are packaged.

Kernel checking cannot establish prose-source fidelity, dependency-binary provenance without a dependency-source rebuild, or literature-search completeness. Cached package binaries remain trusted. The inventory and controls supplement checking and independent review. The ordinary proof above is readable without trusting a search script. No new novelty clearance or acceptance is claimed; root decides and credit remains zero.

8. Related work and access limits

The single-w theorem in [1] uses a scalar Schur complement and path-generating functions. Bickel-Hong [2] develop graph modifications and contact computations while retaining the one-w setting in the relevant results. Their journal Remark 3.2 explicitly preserves the maximum convention. Neither theorem applies to this graph's two w vertices. The possibility of unequal contact data for general stable polynomials in [4] does not itself realize the present unweighted adjacency graph or settle the nearest-colour conjecture.

Body reads are targeted, not cover-to-cover proof audits: [1] Sections 2,4,6 in arXiv v1/v2; [2] Section 3, Remark 3.2, the graph-extension results and the ELA journal body; [3] Definition 2.5 and Theorem 3.1; [4] Theorems 2.12,2.19 and adjacent discussion; [5] local boundary structure; [6] introduction and Remark 1.15; and the contact-related statements in [7,8]. The source journal body remains unread: the excluded ScienceDirect route was not retried.

mathdb:2611.00033v1 04 Nov 2026

9. Bounded prior-art record and bibliography

The independent review initially held prior-art clearance. Root adjudication subsequently changed that gate to qualified pass under a bounded standard. Its recorded Omega head is 440268e282fe97cd7e8d26c931f54f031a839271, with complete D5 and Problems tree screens (13,610 and 1,670 entries). The unrelated word-representable graph hit was distinguished. The review's OpenAI/math head was 4eaee87548250b2682affda7d05d947c05554bae. These are dated preserved screens, not a new live clearance by this manuscript lane and not exhaustive body review. The dossier preserves the registry bodies, review and root ruling.

Knese's work [6], first posted in 2024, actually cites both graph papers in its later version's Remark 1.15. Its inspected discussion describes adjacency constructions without a multiple-w refutation. OpenAlex returned no indexed citing records even though that citation exists; its empty output therefore provides no absence evidence. Search challenge pages, irrelevant results and failed guessed author URLs were recorded as failures. Unread works include Hong's thesis, the admissible-numerators companion paper and other unindexed citing literature. This manuscript does not upgrade those limits into a priority claim. The accessible arXiv source and journal contact definition suffice for the mathematical comparison, while novelty remains subject to root review.

The bibliography below identifies bodies actually inspected in the preserved research record. For [7,8] the relevant sections, rather than complete proofs, were read. Publication metadata is stated only where corroborated in the retained bodies; arXiv identifiers otherwise specify the exact work. No unread journal version of [1] is cited as having been read.

[1] L. Adlin, G. Thai, S. Tiscareno and R. Tully-Doyle. Cauchy transforms of colored graphs in two variables. arXiv:2410.10695, v1 (2024), v2 (2025). Earlier title: Pick functions as Cauchy transforms of graphs. Sections 2,4,6; Theorem 4.6 and Conjecture 6.1.

[2] K. Bickel and Y. Hong. Stable polynomials via undirected colored graphs. Electronic Journal of Linear Algebra 42 (March 2026), 213-238; arXiv:2506.12624. Section 3.1, Remark 3.2 and the one-w extension results.

[3] K. Bickel, J. E. Pascoe and A. Sola. Level curve portraits of rational inner functions. Annali della Scuola Normale Superiore di Pisa, Classe di Scienze (5) 21 (2020), 449-494; arXiv:1803.04666. Definition 2.5 and Theorem 3.1.

[4] K. Bickel, G. Knese, J. E. Pascoe and A. Sola. Local theory of stable polynomials and bounded rational functions of several variables. Annales Polonici Mathematici 133 (2024), 95-169; arXiv:2109.07507v2. Local structure and contact data.

[5] G. Knese. Rational inner functions on the polydisk - a survey. arXiv:2409.14604. Boundary-behaviour discussion and Theorem 0.5.4 in the preserved body.

[6] G. Knese. Boundary local integrability of rational functions in two variables. arXiv:2404.05042. Introduction, main-theorem discussion and Remark 1.15 of the preserved later body.

[7] A. Beslikas and A. Sola. On the membership of two-variable Rational Inner Functions in spaces of Dirichlet-type. arXiv:2512.17388. Preserved primary HTML body, main results and Section 3.

[8] A. Beslikas. Composition operators and Rational Inner Functions on the bidisc: A geometric approach. arXiv:2604.20330. Preserved primary HTML body, Theorem 2.2 and Sections 3.1-3.2. These operator-theoretic statements are background screens, not dependencies of the proof.

mathdb:2611.00033v1 04 Nov 2026