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One integral mapping-class orbit in every rank

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Abstract

For every closed oriented surface of genus $g \geq 2$ and every $r \geq 2$, we construct a representation into $\mathrm{SL}_r(\mathcal{O}_L)$, for a number field $L \subset \mathbb{C}$, whose character has Zariski-dense orbit in the entire complex SL_r character variety under the orientation-preserving mapping class group. The rank-two assertion is due to Coccia–Litt and has further antecedents. For $r \geq 3$, two families of simple curves give a full cotangent certificate at a representation of the form $(A, B, C, I, \ldots, I)$ with A, B diagonal. Their joint residual kernel is eliminated by a symmetric/skew-symmetric Cartan degeneration. Arithmetic parameters are chosen by finitely many open conditions; the countably many nonresonance conditions are imposed only on a complex torus already contained in one arithmetic orbit closure. Classical twist Hamiltonians then force that closure to be the whole variety. The higher-rank construction is presented as new to this project, without a claim of worldwide priority.

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1 Statement, inputs, and antecedents

Let Σ_g be a closed connected oriented surface of genus g. Put

$$\mathcal{R}_{g,r} = \mathrm{Hom}(\pi_1(\Sigma_g), \mathrm{SL}_r(\mathbb{C})), \qquad \mathcal{X}_{g,r} = \mathcal{R}_{g,r} /\!\!/ \mathrm{SL}_r(\mathbb{C}),$$

where the latter is the affine character variety. The group $\mathrm{Mod}(\Sigma_g)$ consists of orientation-preserving mapping classes and acts on $\mathcal{X}_{g,r}$ by precomposition; inner automorphisms of the surface group have no effect on characters. We use $[x, y] = xyx^{-1}y^{-1}$ throughout.

Theorem 1.1. *For every pair of integers $g \geq 2$ and $r \geq 2$, there exist a number field $L \subset \mathbb{C}$ and a homomorphism*

$$\rho : \pi_1(\Sigma_g) \longrightarrow \mathrm{SL}_r(\mathcal{O}_L)$$

such that $\mathrm{Mod}(\Sigma_g) \cdot [\rho]$ is Zariski dense in $\mathcal{X}_{g,r}$.

Here $\mathcal{O}_L$ is the full ring of integers. The conclusion concerns one orbit of an actual integral representation and the entire complex character variety. There is no generic-input condition and no restriction to a marked locus. No assertion for punctured surfaces is made. For $r \geq 3$ the field in our construction depends only on r, although the selected matrices may depend on g.

1.1 The credited rank-two case

Coccia–Litt [2, Propositions 3.1.10 and 5.0.1, Theorem 5.0.2] already prove the assertion for $r = 2$, in a more general relative setting. Their Proposition 5.0.1 produces an actual integral representation that is good for a pants decomposition, and Proposition 3.1.10 proves density of its mapping-class orbit. To meet the unit hypothesis one can use $K = \mathbb{Q}(\sqrt{5})$ and $v = (1 + \sqrt{5})/2$: both v and $v - v^{-1} = 1$ are units, and v is not a root of unity. Their field extension and construction therefore apply to every closed genus $g \geq 2$. Density over $\overline{\mathbb{Q}}$ remains density after extending scalars to $\mathbb{C}$: for each finite-dimensional space of polynomials of bounded degree, the evaluation conditions have coefficients in $\overline{\mathbb{Q}}$, so their kernel commutes with extension of scalars. The remainder of the proof assumes $r \geq 3$.

Litt's survey [6, Theorem 5.4.13] gives another rank-two single-orbit route, using a Shimura-curve local system with a unitary Galois conjugate and Previte–Xia's Theorem 1.4 [12]. Golsefidy–Tamam [4, Section 3.3 and Theorems G–J] identify exceptional loci that must be excluded in the general formulation of that Previte–Xia theorem. We record both the antecedent and this caveat; we do not use the uncaveated statement as an input to our rank-two proof. Golsefidy–Tamam's Theorem I gives a substantially stronger rank-two orbit-closure theorem, under its sufficiently-dense, nonexceptional, infinite-orbit hypotheses. Their detailed orbit-closure preprint [5] explicitly treats SL_2.

1.2 What the higher-rank argument adds

The pinned manuscript [10] proves potential density of the *whole set* of integral characters in every rank, already over one number field. Its closed-surface argument first puts a whole fixed-spectrum fiber in that closure, then uses twist-torus saturation, Baire avoidance, and a submersion. Those operations are antecedents here; density of a union of arithmetic orbits does not itself choose one orbit.

Goldman [3] supplies the symplectic and Hamiltonian twist machinery. Pickrell–Xia [11, Proposition 2.1.24 and Lemma 2.1.34] provide a close antecedent to infinitesimal spanning: centralizers of products hg^n, dense torus powers, and a BCH/root-component argument, followed by transverse sewing in their Section 3. Their result concerns compact groups and

generic fibers in an almost-everywhere argument. The task here is to certify a particular complex family with two commuting handle matrices inside the closure of one arithmetic orbit.

For comparison, the commuting twist tori associated to a pants decomposition have dimension at most $(3g-3)(r-1)$. For $r \geq 3$,

$$(3g-3)(r-1) < (g-1)(r^2-1) = \tfrac{1}{2}\dim \mathcal{X}_{g,r}.$$

Thus the rank-two pants-torus fiber argument does not simply give the required higher-rank cotangent certificate. We use two noncommuting families of simple curves and reduce their common kernel to

$$(\mathrm{Ad}_A - \mathrm{Ad}_C)S = 0, \qquad (I - \mathrm{Ad}_C)S \in \mathfrak{h} + \mathfrak{h}_C. \tag{1.1}$$

A symmetric/skew-symmetric degeneration proves this system generically injective. Arithmetic parameters are then chosen in finitely many nonempty opens. A further complex parameter, already inside the arithmetic orbit closure, is chosen outside countably many resonance sets.

Golsefidy–Tamam [4, Section 2.4, second remark] explicitly suggest extending their results to SL_d and other Chevalley groups using Larsen–Pink. That extension is not proved there. Our existence theorem is a special case of the anticipated broader higher-rank picture, rather than a claim that such a picture was unanticipated. The construction and proof are new to this project; bounded searches found no higher-rank single-orbit integral theorem, but do not establish worldwide novelty.

The choice of point cannot be replaced by an unexamined high-rank dense-image assertion. A MathOverflow comment by M. Kapovich reports non-dense mapping-class orbits of Zariski-dense high-rank unitary representations [9]. We cite that as a discussion pointer, not as a proved theorem used here. Independently, the actual paper of Biswas–Koberda–Mj–Santharoubane [1, Proposition 1.2 and Section 5] constructs infinite-image representations whose GL_n *characters* are fixed by the entire mapping class group, using TQFT representations and point pushing. That is the precise published statement we use for comparison; it is not the same as the stronger dense-image observation in the comment. Neither observation is needed as a lemma in our proof.

1.3 Geometric input and conventions

We use the following exact input from [10]: the determinant-one fiber

$$\{(A_1, B_1, \ldots, A_g, B_g) \in \mathrm{SL}_r^{2g} : \prod_{i=1}^{g} [A_i, B_i] = I\}$$

is irreducible of dimension $(2g-1)(r^2-1)$. This is the SL_r intermediate assertion established *inside the proof* of Proposition `prop:surface-irreducibility`, in `surfaces.tex`, before the scalar-extension paragraph. The proposition's stated conclusion concerns the GL_r fiber; we do not infer determinant-one irreducibility from that conclusion. The continuous surjective quotient morphism then makes $\mathcal{X}_{g,r}$ irreducible.

At a smooth irreducible representation the tangent space of $\mathcal{X}_{g,r}$ is the kernel of the linearized surface relation modulo infinitesimal simultaneous conjugation. The trace pairing on $\mathfrak{sl}_r$ and cup product give the nondegenerate holomorphic Goldman form on this smooth locus [3, Introduction and Sections 2–3]. We will check smoothness at our chosen representations directly. All flow arguments take place in an ambient neighborhood in this smooth irreducible locus, hence in Goldman's D/G; the orbit closure itself need not be smooth.

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Write the first two pairs of surface generators as $(a,b),(c,d_0)$ and the others as (a_i,b_i) for $3\le i\le g$. Their matrices are $(A,B,C,D),(A_i,B_i)$. Let $T\subset \mathrm{SL}_r$ be the diagonal torus and $\mathfrak{h}\subset\mathfrak{sl}_r$ its Lie algebra. Given a regular semisimple $C=QC_\mu Q^{-1}$, write $T_C=QTQ^{-1}$ and $\mathfrak{h}_C=Q\mathfrak{h}Q^{-1}$.

2 An arithmetic torus with dense cyclic generators

The following elementary unit construction fixes the number field before any genericity condition is imposed.

Lemma 2.1. *For every $r\ge 2$ there are a number field $L\subset\mathbb{C}$ and units $\mu_1,\dots,\mu_r\in\mathcal{O}_L^\times$ with product one such that*

$$\prod_i \mu_i^{e_i}=1 \quad\Longleftrightarrow\quad e_1=\cdots=e_r \qquad (e_i\in\mathbb{Z}).$$

In particular, $C_\mu=\mathrm{diag}(\mu_1,\dots,\mu_r)$ generates a Zariski-dense cyclic subgroup of T and has distinct eigenvalues.

Proof. Fix an integer $c_0\ge 3$ and set $f(X)=X^r-c_0X^{r-1}-1$. On $|X|=1$, the middle term has modulus $c_0>2\ge|X^r-1|$. Rouché's theorem gives exactly $r-1$ roots inside the unit circle. Since $f(c_0)=-1$ and $f(x)$ tends to infinity on the positive real axis, there is one real root greater than c_0, the unique root outside. If f were reducible over $\mathbb{Q}$, Gauss's lemma would give a nonconstant monic integral factor not containing that outside root. Its constant term would be a nonzero integer of modulus less than one, a contradiction. Thus f is irreducible; characteristic zero gives distinct roots $\alpha_1,\dots,\alpha_r$. They are algebraic units because f is monic with constant term -1.

Suppose $\prod_i\alpha_i^{e_i}$ is a root of unity. Choose an index whose exponent is $M=\max_i e_i$. Irreducibility allows a Galois automorphism σ of the splitting field that sends this root to the unique outside root. Since $|\prod_i\sigma\alpha_i|=1$, we have

$$0=\sum_i e_i\log|\sigma\alpha_i|=\sum_i(e_i-M)\log|\sigma\alpha_i|.$$

The term for the outside root is zero; all other logarithms are strictly negative and all their coefficients are nonpositive. Every term is therefore nonnegative, and equality forces $e_i=M$ for every i.

Choose a root of unity η with $\eta^r=(-1)^{r-1}$, and put $\mu_i=\eta\alpha_i$. Since $\prod_i\alpha_i=(-1)^{r+1}$, their product is one. A relation among the μ_i gives a root-of-unity relation among the α_i, so its exponents are all equal. Conversely all equal exponents give a relation. Take L to be the splitting field with η adjoined. The character lattice of T is $\mathbb{Z}^r/\mathbb{Z}(1,\dots,1)$. A proper algebraic subgroup of a torus is annihilated by a nontrivial character, so the stated relation lattice proves cyclic density. Distinctness follows already from that of the α_i. □

For a tuple $m=(m_1,\dots,m_{r-1})$ of positive integers define

$$B(m)=\mathrm{diag}\left(\mu_1^{m_1},\dots,\mu_{r-1}^{m_{r-1}},\prod_{i<r}\mu_i^{-m_i}\right). \tag{2.1}$$

Lemma 2.2. *The set of pairs $(B(m),Q)$, with all $m_i\ge 1$ and $Q\in\mathrm{SL}_r(\mathbb{Z})$, is Zariski dense in $T\times\mathrm{SL}_r(\mathbb{C})$. Every $B(m)$ generates T densely. Every $C=QC_\mu Q^{-1}$ belongs to $\mathrm{SL}_r(\mathcal{O}_L)$ and generates T_C densely.*

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Proof. Each μ_i has infinite order, by Lemma 2.1. The first $r-1$ coordinates in (2.1) range independently over infinite subsets of $\mathbb{C}^\times$. A Laurent polynomial vanishing on their product is zero, by induction on the number of variables. This proves density of the $B(m)$. If a character with exponents e_i is one on $B(m)$, then

$$\prod_{i<r} \mu_i^{m_i(e_i-e_r)} = 1.$$

Apply Lemma 2.1 with final exponent zero. All exponents are zero, hence all $e_i = e_r$, proving cyclic density.

The Zariski closure of $\mathrm{SL}_r(\mathbb{Z})$ contains each elementary root group: it contains its integral parameters, an infinite subset of an affine line. Those root groups generate $\mathrm{SL}_r(\mathbb{C})$, proving density. Products of dense sets are dense. Finally Q^{-1} has integral entries because $\det Q = 1$, so $QC_\mu Q^{-1}$ is integral; its torus density follows by conjugation. □

All later parameter opens will be taken inside this single fixed-eigenvalue family. We never assume that a countable set of arithmetic points avoids a countable union of proper closed sets.

3 Two families of simple curves

We fix the based-word convention, since the order of the factors matters both topologically and in the tangent calculation. A genus-two surface with one boundary component is obtained by thickening a rose with cyclic half-edge order

$$a,\ b^{-1},\ a^{-1},\ b,\ c,\ d_0^{-1},\ c^{-1},\ d_0. \tag{3.1}$$

Composing the cyclic permutation with edge reversal gives one boundary cycle with word $[a,b][c,d_0]$. In genus g append the blocks $a_i, b_i^{-1}, a_i^{-1}, b_i$, and cap the boundary to obtain Σ_g. A turn from a letter x to y is drawn as a chord joining half-edges x^{-1} and y in the vertex disk.

Lemma 3.1. *For all $k,l \geq 1$, the words*

$$u_0(k,l) = ba^k d_0 c^l, \tag{3.2}$$
$$u_1(k,l) = aba^{k-1} c d_0 c^{l-1} \tag{3.3}$$

represent simple nonseparating curves. For every $i \geq 3$, so do $u_1(k,l)a_i$ and $u_1(k,l)b_i^{-1}$.

Proof. First consider the words bd_0 and $abcd_0$. Their turn chords in (3.1), numbered from one to eight, are respectively

$$\{2,8\},\{4,6\}; \qquad \{3,4\},\{2,5\},\{7,8\},\{1,6\}.$$

In each case the chords are noninterleaving. Each edge is used once, so these chords and the edge bands smooth to a single embedded closed curve. Its homology class is nonzero and primitive, hence it is nonseparating after capping the boundary.

The oriented Dehn twist about a, or its inverse according to the choice of a positive twist, has based action $b \mapsto ba$ and fixes a and the other generators. This follows by cutting the a-annulus and following the single transverse b-arc across one twist. Iteration gives $b \mapsto ba^m$. The identical construction on the other handle gives $d_0 \mapsto d_0 c^n$. These are actual boundary-fixing twist powers; algebraically $[a, ba^m] = [a,b]$ and $[c, d_0 c^n] = [c, d_0]$, so the

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chosen based boundary word is fixed exactly. Apply exponents (k, l) to bd_0 and $(k-1, l-1)$ to $abcd_0$. This gives (3.2)–(3.3) and preserves simplicity and nonseparation.

For an additional handle let p be the position of a_i in the full cyclic order. The closing chord $\{1, 6\}$ of $abcd_0$ is replaced, for $abcd_0a_i$, by $\{6, p\}, \{1, p+2\}$, and for $abcd_0b_i^{-1}$ by $\{6, p+1\}, \{1, p+3\}$. These chords are again noninterleaving with each other and with the three unchanged chords. Each edge is used once. The resulting curves are simple and have nonzero homology classes. The same boundary-fixing twists give the claimed words. □

We will also use the twist about b whose based action is $a \mapsto ab$. It fixes all other generators and satisfies $[ab, b] = [a, b]$. All twists used below therefore lie in the orientation-preserving mapping class group of the closed surface. No replacement of ba^k by a^kb is implicit.

4 Cartan genericity and a residual linear system

Throughout this section $r \geq 3$, and the eigenvalues μ_i are those fixed in Lemma 2.1. All parameter spaces are over $\mathbb{C}$.

Lemma 4.1. *There is a nonempty Zariski open set of $Q \in \mathrm{SL}_r$ on which both of the following hold:*

(P1) For every $i \neq j$, the map $\mathfrak{h}_C \to \mathbb{C}^2$, $K \mapsto (K_{ij}, K_{ji})$, has rank two.

(P2) The torus T_C contains a regular matrix N whose off-diagonal entries are all nonzero and such that, for every three distinct indices,

$$N_{ij}N_{jk}N_{ki} \neq N_{ik}N_{kj}N_{ji}. \tag{4.1}$$

Proof. For (P1) at the pair $(1, 2)$, take a matrix H whose only nonzero off-diagonal entries are

$$H_{12} = H_{21} = H_{23} = H_{31} = 1;$$

all other off-diagonal entries are zero. Give it generic diagonal entries. The projections of H and H^2 to entries $(1, 2), (2, 1)$ are

$$(1, 1), \qquad (H_{11} + H_{22},\ H_{11} + H_{22} + 1),$$

so their determinant is one. Generic diagonal entries make H regular semisimple (one may first take distinct widely separated real diagonal entries). Subtract the scalar part of H; this does not change the independence, since the change in H^2 is a scalar plus a multiple of H. The two traceless polynomial expressions belong to the eigen-Cartan of H. Choosing an eigenbasis and rescaling one column makes its matrix Q have determinant one. The rank-two condition is an open condition on Q, since it is the nonvanishing of at least one 2×2 minor of a fixed basis of $Q\mathfrak{h}Q^{-1}$. Relabeling the witness treats every pair. The finitely many nonempty opens intersect in the irreducible variety SL_r.

For (P2), the product of the off-diagonal coordinates, the finitely many triangle differences in (4.1), the determinant, and the discriminant is a nonzero polynomial on the full matrix space: each factor is a nonzero polynomial, and that polynomial ring is a domain. Choose a matrix where it is nonzero and scale it to determinant one. Its eigen-Cartan is a witness. More explicitly, substitute $N = Q\,\mathrm{diag}(t_1, \ldots, t_r)Q^{-1}$, $\prod t_i = 1$, in the product of the required factors. This is a Laurent polynomial in $t_1, \ldots, t_{r-1}$ whose coefficients are regular in Q. Its not being identically zero is a nonempty open condition on Q. It is equivalent to the existence of a suitable N. Intersect with (P1). □

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Lemma 4.2 (Principal-minor differential rigidity)**.** *Suppose a matrix $M \in M_r(\mathbb{C})$ has no zero off-diagonal entries and satisfies (4.1) with M in place of N. If a variation $E \in M_r(\mathbb{C})$ makes every principal minor stationary, then $E = [h_0, M]$ for a diagonal matrix h_0.*

Proof. The one-by-one minors give $E_{ii} = 0$. Set $x_{ij} = E_{ij}/M_{ij}$ for $i \neq j$. The two-by-two minors give $x_{ij} + x_{ji} = 0$. Differentiate the three-by-three determinant by the permutation formula. Its diagonal term vanishes because $E_{ii} = 0$; its transposition terms vanish by the pair identities. The remaining two oriented cycles give

$$(M_{ij}M_{jk}M_{ki} - M_{ik}M_{kj}M_{ji})(x_{ij} + x_{jk} + x_{ki}) = 0.$$

The first factor is nonzero, so all oriented triangle sums vanish. Set $h_1 = 0$ and $h_i = x_{i1}$ for $i \neq 1$. Antisymmetry and the triangle with reference index 1 give $x_{ij} = h_i - h_j$ for every $i \neq j$, including the cases involving index 1. Thus $E_{ij} = (h_i - h_j)M_{ij}$, including the diagonal entries. □

This is an infinitesimal form of classical principal-minor rigidity under diagonal similarity. Loewy's global theorem [7] and the pair-product and triangle-cycle identities of Mantelos [8, Sections 3–4 and Lemma 6.2] are antecedents. We give the elementary proof because its exact differential hypotheses are what is needed below; no new contribution is claimed for this lemma alone. In rank three the three-by-three minor is the full determinant; in both applications its derivative vanishes automatically.

Proposition 4.3. *There is a nonempty Zariski open subset of $T \times \mathrm{SL}_r$ such that, for $C = QC_\mu Q^{-1}$, the only $S \in \mathfrak{sl}_r$ satisfying*

$$(\mathrm{Ad}_A - \mathrm{Ad}_C)S = 0, \qquad (I - \mathrm{Ad}_C)S \in \mathfrak{h} + \mathfrak{h}_C \tag{4.2}$$

is zero, and $\mathfrak{h} \cap \mathfrak{h}_C = 0$.

Proof. Choose a real orthogonal matrix Q of determinant one with every entry nonzero. Such matrices exist: each individual coordinate is nonzero on an open dense subset of the real analytic manifold $\mathrm{SO}(r)$, and a finite intersection is nonempty. Every member of $\mathfrak{h}_C$ is then a complex symmetric matrix. Moreover $\mathfrak{h} \cap \mathfrak{h}_C = 0$: if $\mathrm{diag}(k)Q = Q\,\mathrm{diag}(\lambda)$, nonzero Q_{ij} implies $k_i = \lambda_j$ for all i, j, so both diagonal matrices are scalar, and tracelessness makes them zero.

Choose a real traceless diagonal H with distinct entries, and put $A(t) = \exp(tH)$ for real t. Suppose there were nonzero solutions of (4.2) for a sequence $t \to 0$, $t \neq 0$. Normalize the Hermitian norms of S_t to one and pass to a convergent subsequence $S_t \to K$. The first equation gives $K \in \mathfrak{h}_C$. It also gives

$$\frac{(I - \mathrm{Ad}_C)S_t}{t} = \frac{(I - \mathrm{Ad}_{A(t)})S_t}{t} \longrightarrow -[H, K].$$

The left-hand side lies in the fixed closed subspace $\mathfrak{h}+\mathfrak{h}_C$, hence $[H, K]$ is complex symmetric. Since H and K are symmetric, their commutator is skew-symmetric. Therefore $[H, K] = 0$. Distinct entries of H make K diagonal, whence $K \in \mathfrak{h} \cap \mathfrak{h}_C = 0$, contradicting its norm one. Thus for all sufficiently small nonzero t the system is injective.

For global openness use the augmented linear map

$$\begin{aligned} &\mathfrak{sl}_r \oplus \mathfrak{h} \oplus \mathfrak{h} \longrightarrow \mathfrak{sl}_r \oplus \mathfrak{sl}_r, \\ &\quad (S, \alpha, \beta) \longmapsto \big((\mathrm{Ad}_A - \mathrm{Ad}_C)S,\ (I - \mathrm{Ad}_C)S - \alpha - Q\beta Q^{-1}\big). \end{aligned} \tag{4.3}$$

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It is injective exactly when (4.2) has no nonzero solution and $\mathfrak{h} \cap \mathfrak{h}_C = 0$. Indeed a vector in its kernel gives the residual system; after $S = 0$, the remaining equality is exactly an intersection of the two Cartans. Conversely each residual solution admits such α, β. All entries of (4.3) are regular functions on $T \times \mathrm{SL}_r$. Injectivity is a maximal-minor condition, and the witness just constructed proves its nonemptiness. □

Lemma 4.4. *There is a nonempty open set of triples $(A, B, Q) \in T \times T \times \mathrm{SL}_r$ satisfying (P1), (P2), Proposition 4.3, and the following additional conditions: B has distinct diagonal entries; every entry of Q and $C = QC_\mu Q^{-1}$ is nonzero; and for each fixed cardinality $1 \le j < r$, the products*

$$a_I = \prod_{i \in I} a_i \qquad (|I| = j), \quad A = \mathrm{diag}(a_1, \dots, a_r),$$

are distinct for distinct subsets I.

Proof. Each condition is nonempty and open, and the parameter space is irreducible. For the subset products, $\mathbf{1}_I - \mathbf{1}_J$ is not a multiple of $(1, \dots, 1)$ when $I \neq J$ have equal size, so the forbidden character equality is proper. Nonzero entries of Q and regularity of B are elementary open conditions. Each off-diagonal entry of a matrix in the regular conjugacy class of C_μ is nonzero somewhere: conjugate a diagonal matrix by an elementary matrix and use two distinct eigenvalues. Each diagonal entry is nonzero somewhere, since the eigenvalues themselves are nonzero. Their individual nonempty opens therefore intersect. The other conditions are covered by the preceding results. □

5 The two-band cotangent certificate

For a matrix $M \in \mathrm{SL}_r$ let $e_j(M)$ be the jth elementary symmetric function of its eigenvalues, so $\det(zI - M) = \sum_{j=0}^{r} (-1)^j e_j(M) z^{r-j}$. We use $1 \le j < r$; $e_r = 1$ is constant. For a group word w, these are regular invariant functions on the representation variety and descend to its character variety.

The use of centralizers of twisted products and their powers has a close precedent in Pickrell–Xia [11, Proposition 2.1.24, Lemma 2.1.34]. Here we need the following explicit calculation at a family whose first two handle matrices commute; no generic compact-group spanning theorem is substituted for the calculation.

Proposition 5.1. *Let (A, B, Q) satisfy Lemma 4.4, and set $C = QC_\mu Q^{-1}$. At the genus-two representation (A, B, C, I), the common tangent kernel of the characteristic coefficients of A, C and of all the simple curves $u_0(k, l), u_1(k, l)$, $k, l \ge 1$, consists exactly of infinitesimal simultaneous conjugations. The resulting character is a smooth irreducible point, and these differentials span its cotangent space.*

Proof. Since A, C are regular semisimple, stationarity of their characteristic coefficients says that their variations are tangent to their conjugacy classes. Subtract an infinitesimal simultaneous conjugation to write a common-kernel tangent vector as

$$\delta A = 0, \qquad \delta B = BU, \qquad \delta C = [Z, C], \qquad \delta D = V, \tag{5.1}$$

with $U, V, Z \in \mathfrak{sl}_r$. The choice of Z is allowed to change by an element of $\mathfrak{h}_C$.

The first band. Cyclic rotation of a product of invertible matrices preserves its characteristic polynomial. Use the cyclic representative $A^k D C^l B$ for u_0. Its derivative is $A^k E_l$, where

$$E_l = (V + Z) C^l B + C^l (\mathrm{Ad}_B U - Z) B. \tag{5.2}$$

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For any M and any j,

$$e_j(A^k M) = \sum_{|I|=j} a_I^k \det M[I, I]. \tag{5.3}$$

For fixed $j < r$ the distinct nonzero numbers a_I make the coefficient matrix for $k = 1, \dots, \binom{r}{j}$ a nonsingular Vandermonde matrix times a nonsingular diagonal matrix. Thus stationarity for all $k \geq 1$ forces every $j \times j$ principal minor of $M = C^l B$ to be stationary under E_l. The full determinant is stationary too: the variations come from products in SL_r.

Positive powers of C are Zariski dense in T_C. To see that positive powers suffice, their Zariski closure in the connected torus is the same as the closure of the full cyclic group; equivalently a Laurent polynomial vanishing on all positive powers also vanishes on all integer powers by the recurrence satisfied by its finitely many exponential terms. Therefore the resulting regular identities hold with C^l replaced by every $N \in T_C$. Put

$$X = V + Z, \qquad Y = \mathrm{Ad}_B U - Z, \qquad W = X + Y = V + \mathrm{Ad}_B U.$$

All principal minors of NB are stationary for the variation $(XN + NY)B$. At $N = I$, the one-by-one identities give $W_{ii} = 0$. Differentiate the stationary two-by-two identity along $N = I + tK + O(t^2)$, where $K \in \mathfrak{h}_C$. The one-by-one stationary identities hold for every N, so the derivative of the diagonal-product part is zero. The two off-diagonal terms give, after dividing by $-B_{ii}B_{jj} \neq 0$,

$$K_{ij}W_{ji} + K_{ji}W_{ij} = 0. \tag{5.4}$$

Condition (P1) forces $W_{ij} = W_{ji} = 0$ for each pair. Hence $W = 0$ and $V = -\mathrm{Ad}_B U$, and the variation reduces to $[X, N]B$.

Choose a regular N as in (P2). The matrix NB has nonzero off-diagonal entries, and each triangle difference is that of N multiplied by the nonzero scalar $B_{ii}B_{jj}B_{kk}$. Lemma 4.2 gives

$$[X, N]B = [h_0, NB] = [h_0, N]B$$

for a diagonal h_0. Thus $X - h_0$ centralizes regular N, whose centralizer in SL_r is T_C. Subtracting scalar parts makes h_0 traceless and $X - h_0 \in \mathfrak{h}_C$. Since $X = Z - \mathrm{Ad}_B U$, subtracting the simultaneous diagonal conjugation by h_0 yields precisely

$$\delta A = 0, \qquad \delta B = SB, \qquad \delta C = [S, C], \qquad \delta D = -S, \qquad S = \mathrm{Ad}_B U. \tag{5.5}$$

Here diagonal conjugation does not change A or B, and the remaining part of $Z - h_0 - S$ commutes with C.

The derivative of the surface relation in (5.5) is

$$(\mathrm{Ad}_A S - S) + (S - \mathrm{Ad}_C S) = (\mathrm{Ad}_A - \mathrm{Ad}_C)S.$$

Consequently any relation tangent in this gauge satisfies

$$T_0 := \mathrm{Ad}_A S = \mathrm{Ad}_C S. \tag{5.6}$$

The second band. At the base point $u_1(k, l)$ has matrix $M = A^k BC^l$. Differentiate the actual word $ABA^{k-1}CDC^{l-1}$ in the gauge (5.5). The B contribution is $T_0 M$. For the last three factors one has

$$\delta(CDC^{l-1}) = [S, C^l] - CSC^{l-1}.$$

Using $CS = T_0 C$ gives

$$\begin{aligned} \delta M &= T_0 M + A^k B(S - T_0)C^l - MS \\ &= [T_0, M] + A^k B[R, C^l], \qquad R = S - \mathrm{Ad}_C S. \end{aligned} \tag{5.7}$$

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The conjugation term has zero characteristic derivatives. Apply (5.3) and density of the powers of C once more. All principal minors of BN are stationary for the variation $B[R, N]$, for every $N \in T_C$. The full determinant is stationary because $\operatorname{tr}(N^{-1}[R, N]) = 0$. At a matrix from (P2), Lemma 4.2 yields

$$B[R, N] = [h_1, BN] = B[h_1, N],$$

so $R - h_1 \in \mathfrak{h}_C$ after removing scalar parts. Hence $(I - \mathrm{Ad}_C)S \in \mathfrak{h} + \mathfrak{h}_C$. Combined with (5.6), this is exactly (4.2). Proposition 4.3 forces $S = 0$. Undoing the gauges proves the kernel assertion. The converse holds because all the functions are conjugation invariant.

Smoothness and the quotient. Since B has distinct diagonal entries, a B-invariant subspace is a sum of coordinate lines. No proper nonzero such sum is invariant under C, because every entry of C is nonzero. The representation is irreducible. The image of the derivative of the relation contains

$$\operatorname{im}(\mathrm{Ad}_A - I) + \operatorname{im}(\mathrm{Ad}_C - I),$$

by left variations of B and variations of D. Under the nondegenerate trace pairing its annihilator is the common Lie centralizer $\mathfrak{h} \cap \mathfrak{h}_C = 0$. The relation map is therefore submersive. The relation variety is smooth of dimension $3(r^2-1)$ at the point; simultaneous conjugation has dimension $r^2 - 1$. Its stabilizer is the scalar center, which acts trivially. The standard irreducible smooth quotient description from Section 1.3 gives a smooth character point of dimension $2(r^2 - 1)$, with tangent space the quotient by conjugations. The common-kernel assertion is thus equivalent to cotangent spanning. □

Proposition 5.2. *For every $g \geq 2$, at the representation*

$$\rho_A = (A, B, C, I, I, \ldots, I) \tag{5.8}$$

with the same open conditions, the functions in Proposition 5.1, together with the traces of $u_1(k, l)a_i$ and $u_1(k, l)b_i^{-1}$ for $i \geq 3$, span the full cotangent space of $\mathcal{X}_{g,r}$. A finite list of these functions suffices.

Proof. The commutator of an extra pair of identity matrices has zero first derivative, so its two tangent variables $\delta A_i, \delta B_i \in \mathfrak{sl}_r$ are unrestricted by the linearized relation. Infinitesimal simultaneous conjugation is zero on these identity entries. Apply the core argument to a common-kernel tangent and gauge its first four variations to zero. The trace differentials of the additional curves then are exactly

$$\operatorname{tr}(U_{kl}\delta A_i), \qquad -\operatorname{tr}(U_{kl}\delta B_i), \qquad U_{kl} = A^k B C^l. \tag{5.9}$$

No subtraction of a core trace is necessary: its derivative is already zero in this gauge.

The matrices U_{kl} for $1 \leq k, l \leq r$ span all of $M_r(\mathbb{C})$. Indeed $A, A^2, \ldots, A^r$ span the diagonal matrices by Vandermonde interpolation. For fixed row i,

$$\operatorname{row}_i(C^l) = \sum_{j=1}^{r} \mu_j^l Q_{ij} \operatorname{row}_j(Q^{-1}).$$

All Q_{ij} are nonzero and the μ_j are distinct, so varying $l = 1, \ldots, r$ spans all row vectors. The additional nonzero factor B_{ii} does not change the span. The trace pairing against all of M_r detects every tangent matrix, so (5.9) kills all extra variations. Smoothness follows by the same relation submersion and irreducibility argument as before. The quotient dimension is $(2g - 2)(r^2 - 1)$.

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A spanning set in a finite-dimensional cotangent space has a finite spanning subset. Choose one such subset at one point in the nonempty open parameter set, and let $\mathcal{F}$ be the finite set of underlying simple curves. Retain the required coefficient index for each chosen function; we may add any of the core functions used in constructing this list. No effective bound on this finite list is asserted. □

6 Arithmetic selection before complex nonresonance

The order of choices is essential. We first choose the finite list $\mathcal{F}$ from Proposition 5.2 at one suitable complex parameter triple. We then choose integral B, Q by finitely many open conditions. Only after their choice do we impose nonresonance at a complex A.

Lemma 6.1. *For the fixed finite list $\mathcal{F}$, there is a nonempty open set $\mathcal{U} \subset T \times \mathrm{SL}_r$ of parameters (B, Q) with the following properties:*

(i) On a nonempty open subset of the A-torus, ρ_A is a smooth irreducible character point and the selected differentials span its cotangent space.

(ii) For each band curve in $\mathcal{F}$, its characteristic-coefficient map from the A-torus to T/W is dominant, where W is the symmetric group.

Proof. Put $q = r^2 - 1$. Use a fixed basis of the Lie algebra to trivialize the tangent bundles of the matrix factors. At the initial suitable point, the linear map consisting of the relation derivative together with the selected invariant derivatives has kernel the q-dimensional conjugation space. Its rank is therefore $2gq - q$. A corresponding minor is nonzero. All entries are regular functions of (A, B, Q): inverses are regular on SL_r, and all words are finite. If quotient charts were used instead, one could clear their nonzero denominators with the same conclusion. Include minors or products expressing relation submersion and the explicit irreducibility conditions. This produces a nonzero regular function on $T \times T \times \mathrm{SL}_r$ that guarantees (i). Expand it as a Laurent polynomial in A. The locus of (B, Q) where at least one coefficient is nonzero is a nonempty open; there it is nonzero on a nonempty open of the A-torus.

Along (5.8), each band in $\mathcal{F}$ has the same characteristic polynomial as $A^k B C^l$ for some $k, l \geq 1$. This includes u_0 by cyclic rotation and commutation of A with B; for the extra-handle curves the extra matrix is the identity. At $Q = I$ the map in question is

$$T \longrightarrow T \longrightarrow T/W, \qquad A \longmapsto A^k B C_\mu^l \longmapsto (e_1, \ldots, e_{r-1}).$$

The first arrow is finite surjective and the second is the finite Weyl quotient. In characteristic zero its differential has full rank somewhere. One such Jacobian minor is a Laurent polynomial in A, regular in B, Q. The condition that it is not identically zero in A is nonempty open in (B, Q) and implies dominance. This witness keeps exactly the fixed eigenvalues μ_i; no change of conjugacy class is being made. There are only finitely many curves, so their opens and the open in (i) intersect in the irreducible parameter space $T \times \mathrm{SL}_r$. □

Choose $(B, Q) \in \mathcal{U}$ with $B = B(m)$ and $Q \in \mathrm{SL}_r(\mathbb{Z})$ by Lemma 2.2, and fix $C = QC_\mu Q^{-1}$. Then

$$\rho_0 = (I, B, C, I, I, \ldots, I) \tag{6.1}$$

is an actual representation into $\mathrm{SL}_r(\mathcal{O}_L)$: every commutator is identity. Let

$$Z = \overline{\mathrm{Mod}(\Sigma_g) \cdot [\rho_0]}^{\mathrm{Zar}} \subset \mathcal{X}_{g,r}.$$

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The twist $a \mapsto ab$ has powers sending ρ_0 to $(B^n, B, C, I, \dots, I)$. Since B generates T densely, continuity of the quotient morphism gives

$$[\rho_A] \in Z \qquad \text{for every } A \in T. \tag{6.2}$$

Lemma 6.2. *There is an $A \in T$ such that $x = [\rho_A]$ is a smooth irreducible character point where the selected differentials span the cotangent space, and every curve in $\mathcal{F}$ has regular semisimple monodromy whose powers are Zariski dense in its centralizer torus.*

Proof. In a diagonal torus, failure of cyclic density is equivalent to $\chi(t) = 1$ for some nontrivial character $\chi \in \mathbb{Z}^r/\mathbb{Z}(1, \dots, 1)$. For each such character its kernel is a proper closed subset of T; the finite Weyl image of the union of its conjugates is a proper closed subset of T/W. Properness follows from dimension, since the quotient is finite. There are countably many such characters. Also remove the discriminant locus in T/W explicitly. For each band map, dominance from Lemma 6.1 makes all these inverse images proper closed subsets of the A-torus. For the curve a the spectral map of A itself is dominant. For the curve c, regular semisimplicity and nonresonance already follow from Lemma 2.1, independently of A.

Intersect the nonempty open from Lemma 6.1(i) with the complements of this countable collection of proper closed sets, for all curves in the finite list. Such an intersection is nonempty by the Baire theorem: on any small closed ball contained in the open smooth torus, proper complex algebraic subsets are closed with empty interior. The selected A has every required property, including avoidance of the discriminant for each band. □

The A of Lemma 6.2 is not asserted to be arithmetic. This is harmless, because (6.2) places it inside the closure of the orbit of the single integral representation (6.1). Neither an arithmetic Baire argument nor a simultaneous Hilbert-irreducibility assertion is used.

7 Twist saturation and an open orbit closure

We complete the proof while allowing the orbit closure to be singular. The torus saturation, Baire passage, and ambient submersion have antecedents in [10, Section on closed-surface dynamics]. Here all of them occur inside the closure of *one* integral orbit.

Let Y be the closure of the image of the A-torus in (6.2). It is irreducible. Choose an irreducible component Z_0 of Z containing Y. For each $\gamma \in \mathcal{F}$, some positive power $\tau_\gamma^{n_\gamma}$ of its Dehn twist preserves Z_0, because each mapping class permutes the finitely many irreducible components of Z.

Lemma 7.1. *In a neighborhood of x in Z_0, points whose monodromy on every $\gamma \in \mathcal{F}$ generates its regular centralizer torus are dense in the complex analytic topology.*

Proof. Intersect Z_0 with the ambient smooth irreducible-character locus and with the open where every γ-monodromy is regular semisimple. This is a nonempty irreducible open Z_0' containing x. For one curve γ, pull back the eigenvalue-ordering cover $T^{\mathrm{reg}} \to (T/W)^{\mathrm{reg}}$ to Z_0'. It is finite étale. Each irreducible component of this cover dominates Z_0': flatness implies that its generic point lies over the generic point of the integral base. Finiteness makes its image closed, so each component in fact surjects onto Z_0'.

For any nontrivial character of T, its being one is a closed condition on the cover. It cannot hold identically on any component, because every component has a point over x, and every ordering of the eigenvalues at x is nonresonant. Its zero locus in each component is therefore proper; under a finite morphism its image is closed and still proper, by dimension.

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There are finitely many covering components and countably many characters. Repeating this for the finite list of curves gives a countable collection of proper closed subsets of Z_0'.

Their complement is dense analytically. For clarity, apply the Baire theorem on small compact balls in the smooth dense locus of the irreducible variety Z_0', where each proper algebraic subset has empty interior; then take closure to include singular points. This also shows density in every relative analytic neighborhood of x. □

Lemma 7.2. *At each point y in the dense set of Lemma 7.1, the centralizer-torus twist orbit about every $\gamma \in \mathcal{F}$ lies in Z_0.*

Proof. Cut the surface along the simple curve γ. For a nonseparating curve the surface group is an HNN extension of the cut-surface group. Holding the cut representation fixed, one may multiply a transverse gluing matrix by any element of the centralizer of $\rho_y(\gamma)$. This is an algebraic torus action on the gluing parameter and a regular map from that torus to the character variety. The Dehn twist is the parameter $\rho_y(\gamma)$ or its inverse, according to orientation. All curves in our list are nonseparating; the analogous separating construction would conjugate one cut component.

The orbit under $\tau_\gamma^{n_\gamma}$ lies in Z_0. Powers of $\rho_y(\gamma)^{n_\gamma}$ are still dense in its connected torus: a character trivial on these powers would give a finite-order value on $\rho_y(\gamma)$, whose suitable multiple would contradict its full cyclic density. The inverse image of the closed set Z_0 under the torus map is closed and contains these dense powers, so it is the whole torus. □

We next specify the complex Hamiltonian convention. On an ambient neighborhood U of x contained in the smooth irreducible-character locus D/G, let ω be the holomorphic symplectic form from the complex trace pairing. For a holomorphic invariant function h_γ let its holomorphic Hamiltonian field be defined by

$$\omega(X_{h_\gamma}, \cdot\,) = \mathrm{d}h_\gamma.$$

The real two-form $\Omega = \Re\omega$ is nondegenerate: if $\Re\omega(v, w) = 0$ for all real tangent vectors w, also test iw to obtain the imaginary part. Under the identification of real tangent vectors with their $(1,0)$ components, the real Hamiltonian of $\Re h_\gamma$ has component X_{h_γ}, and the Hamiltonian of $\Im h_\gamma$ has component $-iX_{h_\gamma}$. Indeed these equalities follow directly from $\Re(-i\mathrm{d}h) = \Im\mathrm{d}h$. Choosing $2\Re\omega$ and doubling the real Hamiltonians gives the same fields.

This is also the variation relation of Goldman [3, Lemma 1.11]. His Theorem 4.3, proved in Theorems 4.5 and 4.7 (the HNN formula (4.6) in the nonseparating case), identifies these real Hamiltonians with centralizer twists. The gradient of an invariant function belongs to the centralizer Lie algebra. Therefore X_{h_γ} is tangent to the complex centralizer-torus orbit wherever the monodromy is regular semisimple. This passage uses both real and imaginary parts and takes place entirely in the ambient smooth locus; it does not assume smoothness of Z_0.

Lemma 7.3. *Every selected holomorphic Hamiltonian field has local complex-time flows that preserve $Z_0 \cap U$, including through its singular points.*

Proof. Let X be one such field and let f be a holomorphic function on a small open subset of U vanishing on Z_0. At a point y from Lemma 7.1, Lemma 7.2 and the Hamiltonian-twist identification give $(Xf)(y) = 0$. By analytic continuity and the density proved there, Xf vanishes on all of Z_0 in that open subset.

The radical ideal sheaf of the reduced analytic subspace $Z_0 \cap U$ is coherent. Near any point choose finite generators $f_1, \ldots, f_s$. The preceding vanishing says that Xf_j lies in that

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ideal sheaf; after shrinking the neighborhood write

$$Xf_j = \sum_{k=1}^{s} a_{jk} f_k$$

with holomorphic coefficients. Along an integral curve of X, the vector $(f_1, \dots, f_s)$ satisfies a homogeneous linear differential equation. With zero initial values its unique solution is zero. Holomorphic ODE existence and uniqueness, or the corresponding real ODE in each complex time direction, shows that the local flow preserves the subspace even when the starting point is singular on Z_0. □

Proof of Theorem 1.1 for $r \geq 3$. At x, the chosen invariant differentials span the ambient cotangent space by Lemma 6.2. Nondegeneracy of ω makes their Hamiltonian vectors span the ambient tangent space. Select a basis of $d = (2g-2)(r^2-1)$ such vectors and compose their local holomorphic flows, starting at x, with complex time parameters $(t_1, \dots, t_d)$ near zero. All intermediate points lie in Z_0 by Lemma 7.3. The derivative of this map at the origin has the selected basis as its columns, and is an isomorphism onto $T_x\mathcal{X}_{g,r}$. The holomorphic inverse function theorem puts a nonempty ambient analytic open subset of $\mathcal{X}_{g,r}$ inside Z_0. Since $\mathcal{X}_{g,r}$ is irreducible by the exact source input in Section 1.3, such an open is Zariski dense. Thus $Z_0 = Z = \mathcal{X}_{g,r}$. The integral seed (6.1) proves the assertion. The already credited rank-two case completes all g, r in the statement. □

A Proof scope and supporting artifacts

The proof of Theorem 1.1 covers every $g \geq 2$ and $r \geq 2$ with the quantifiers stated there. Rank two is imported with explicit credit; for $r \geq 3$ Sections 2–7 give the construction and proof. Classical surface-character geometry and Goldman's Hamiltonian theorem are cited inputs. The irreducibility input is the determinant-one intermediate assertion of the pinned manuscript, identified in Section 1.3. The proof neither classifies all arithmetic orbits nor asserts analytic density of the selected orbit.

A.1 Limited algebraic formal support

A separate Lean artifact, `t027o-formal-v1`, supports a precisely bounded part of Lemma 4.2. Its theorem `triangle_coboundary`, over an arbitrary field and an index type with a chosen reference index, proves that a zero-diagonal antisymmetric array with vanishing oriented triangle sums is of the form $x_{ij} = h_i - h_j$. Its theorem `principal_minor_rigidity` assumes nonzero off-diagonal entries, zero diagonal variation, the explicit pair stationarity polynomials, the six-term cycle derivative polynomials, and unequal forward and reverse triangle products. It concludes $E_{ij} = (h_i - h_j)M_{ij}$.

Those formal hypotheses are scalar polynomial identities. The pair identity becomes the negative two-by-two minor derivative after the diagonal identities; the cycle identity becomes the three-by-three minor derivative only after the diagonal and pair identities. The artifact does *not* prove that they are the derivatives of the corresponding determinant functions. It does not formalize generic Cartan choices, Vandermonde interpolation, the surface curves, the residual degeneration, arithmetic integrality, torus density, Goldman geometry, the analytic ideal argument, or orbit density. It is not a formal proof of Theorem 1.1. Its controls include a generic rational example and a counterexample to deleting the triangle-genericity hypothesis. This counterexample shows that genericity cannot simply be deleted; it does not prove necessity of every triple condition in larger index sets. An independent review adds a formal derivative bridge and further controls in its own file, `T027OReview.lean`;

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those additions are distinct from the author package and still do not cover the geometric orbit argument. The exact coverage and trusted build boundary are documented in its `t027o-formal-v1/README.md`; the manuscript packet supplies an external-path crosswalk for local use. Formal review and release status are kept in the project records, not inferred from the existence of this support file.

A.2 Attribution and reproducibility limits

The arithmetic unit construction, principal-minor cycle identities, Vandermonde arguments, torus saturation, Baire theorem, coherent-ideal argument, and inverse function theorem are supporting tools. They are not separate novelty claims. The higher-rank contribution presented here is the connected construction consisting of a single arithmetic seed, the two-band tangent certificate at its complex torus family, and the arithmetic-first, complex-second parameter choice. The literature comparison in Section 1 is bounded, with no claim of complete priority clearance.

The source revision used is `fd4aeeb2ee4fc729c18d98444fed42fd0529eeeb` of OpenAI's math repository. The author packet records exact source identities and reading limits. Private copies of third-party bodies are excluded from the public source bundle. The accompanying build script recompiles this article from a copied source directory and records source and PDF hashes. These hashes and finite diagnostic computations establish identity or finite checks; they do not replace the mathematical proof.

Acknowledgment of assistance. The argument was developed with AI-assisted calculations, source comparison, and independent project review. Author-side checks included the unit-independence argument, the based-word conventions, the second-band derivative, the residual symmetric/skew witness, and the spectral-cover closure argument. Responsibility for the exposition and its stated limitations remains with the named author. No institutional affiliation is asserted.

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